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Book Reviews

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Featured Review: Diffusion and Ecological Problems: Modern Perspectives. Second Edition. *By Akira Okubo and Simon Levin.* Springer-Verlag, New York, 2001. $59.95. xx+467 pp., hardcover. ISBN 0-387-98676-6.

This is a revised and expanded version of Okubo's classic 1980 text *Diffusion and Ecological Problems: Mathematical Models* [2]. While building on the original text, the revision has brought the subject well into the 21st century: the length has almost doubled and the number of references has tripled; themes hinted at in the original version have been developed and matured. Before his death, Okubo asked that his notes on book changes and expansions be left to a close colleague, Simon Levin, and that "he would know what to do with them." Well, Levin did know what to do with them. The result is an excellent book on the role of diffusion theory in modern spatial ecology. In addition to Okubo's original book and its revisions, the contents have been fleshed out by 12 additional authors, all experts in spatial ecology.

This book occupies the middle ground between mathematical theory and ecological theory. It contains many innovative and original models, some analytical results, but no theorems. However, mathematicians and quantitative biologists alike will find the book a useful guide to the formulation and analysis of diffusion-based models in ecology. While biological applications are discussed, the main focus is not on methods for relating model output directly with experimental or field measurement. By contrast, a text that more fully develops the interplay between diffusion models and experiment is Turchin [3].

The original text grew from Okubo's interest in understanding ecological systems with the tools and background of a mathematical physicist. In this original text, the details of diffusion in the atmosphere and ocean are laid out beautifully, as are the variety of different stochastic and deterministic modeling approaches for describing diffusive processes. Later chapters of the original text include examples of animal diffusion, dynamics of animal grouping, movements in home ranges, patchy distributions of organisms, pattern formation, traveling waves, and the effect of community interactions (e.g., competition predation and so forth) on the spatial distribution of animals. In the revised edition it is these later chapters of the book that have been expanded the most to include new advances, such as receptor kinetics-based taxis, evaluation of diffusion models as a standard tool in animal ecology, continuum approximations for animal grouping, data on home range movements of animals, microscale patchiness in plankton systems, and advances in modeling critical domain size problems, to name a few.

If the reader is looking for a comprehensive review of spatial ecology, he or she will likely put this book down disappointed. Many important contributions to spatial ecology, such as cellular automata, interacting particle systems, lattice or integrodif-

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this book highly as an essential reference for any researcher in probability and statistics.

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Theoretical Numerical Analysis: A Functional Analysis Framework. *By Kendall Atkinson and Weimin Han.* Springer-Verlag, New York, 2001. $59.95. xvi+450 pp., hardcover. ISBN 0-387-95142-3.

Reviewing a text in mathematical science is always a most serious undertaking. The present task is no exception. Indeed, this book is close to me in many ways, among them:

(i) One of the series editors is my colleague at the University of Houston, and one of its advisors took my class on computational control.

(ii) My own research makes a systematic use of tools discussed in this book (as shown, for example, in [1]).

(iii) Some of our previous work ([58] and [59] in the book) had some influence on Chapter 10, and my Ph.D. thesis was on the numerical solution of some nonlinear integral and integro-differential equations by Galerkin methods.

As I see it, the main goal of this book is to provide students and practitioners with basic tools from functional analysis and Sobolev space theory. With these tools, they will be able to address the numerical solution of functional equations and inequalities by approximation and iterative methods, and to investigate the convergence of these methods. This book reflects, clearly, the respective interests of the two authors, namely, the numerical solution of *integral equations* for Atkinson, and of *variational inequalities* for Han. This combination is fortunate since it leads to a text blending topics rarely seen in the same book. The book's contents are as follows:

Chapter 1 is dedicated to *linear (vector) spaces*, with particular attention being paid to *Banach, Hilbert,* and C^k *spaces*.

Chapter 2 is an introduction to the "Theory of Linear Operators on Normed Spaces." It includes important topics such as the open mapping theorem, the principle of uniform boundedness, the Hahn–Banach and Riesz representation theorems, compact operators, and the Fredholm alternative (with applications to Fredholm integral equations).

Chapter 3 is an introduction to "Approximation Theory." As such, it contains classical results on polynomial and trigonometric interpolations, best approximation in Hilbert spaces, and a brief introduction to the minimization of lower semicontinuous functionals over convex sets in Banach spaces.

Chapter 4 is dedicated to the iterative solution of nonlinear equations in Banach spaces. It begins with Banach's fixed-point theorem and various applications to the iterative solution of nonlinear equations in one space dimension, of linear systems, of linear and nonlinear Fredholm and Volterra integral equations of the second kind, and of ordinary differential equations in Banach spaces. In order to prepare the reader for iterative methods such as Newton's, Chapter 4 includes a brief introduction to "Differential Calculus for Nonlinear Operators in Banach Spaces," with a discussion of Fréchet and Gâteaux derivatives. The above introduction is followed by a discussion of Newton's method and its convergence properties. Newton's method is then applied to the solution of nonlinear Fredholm integral equations of the second kind and of nonlinear two-point boundary value problems. The chapter concludes with the Brouwer and Schauder fixed point theorems and with a discussion of the conjugate gradient method for the solution of linear problems for self-adjoint operators in Hilbert spaces.

Chapter 5 is an introduction to finite difference methods, with application to relatively simple parabolic equations such as the heat equation in one space variable. The backward Euler, Crank–Nicolson, and Lax–Wendroff schemes are discussed (directly in the text or as exercises). After these preliminary considerations, the authors take a more general approach and prove the Lax equivalence theorem (i.e., for a consistent difference scheme, stability is equivalent to convergence) for a general class of linear initial value problems.

Chapter 6 is an introduction to Sobolev spaces. The authors manage to avoid the use of distribution theory, through a "direct" definition of weak derivatives. This being done, they define the Sobolev spaces $W^{k,p}(\Omega)$ for k a nonnegative integer and $1 \leq p \leq +\infty$, and prove that $W^{k,p}(\Omega)$ is a Banach space for a well-chosen norm. From there, other spaces are considered, such as $W_0^{k,p}(\Omega)$, $W^{s,p}(\Omega)$ with s a nonnegative real number and $1 \leq p < +\infty$, $W_0^{s,p}(\Omega)$, $W^{-s,p'}(\Omega)$ as the dual space of $W_0^{s,p}(\Omega)$ (here $p' = p/(p-1)$), and finally, $W^{s,p}(\partial\Omega)$, $\partial\Omega$ being the boundary of Ω. Next, the authors give a collection of results concerning the density of smooth function spaces in the Sobolev spaces, the extension properties of Sobolev space functions, the celebrated Sobolev embedding theorem, and, finally, various trace theorems.

When solving partial differential equations in Sobolev spaces, some norms may play a privileged role; this observation leads the authors to mention some classical results concerning norm equivalence, including the celebrated Korn's inequality, so useful in mathematical elasticity (see, e.g., [2]).

When $\Omega = \mathbb{R}^d$, it is possible to define the Sobolev spaces $W^{k,2}(\mathbb{R}^d)$ $(= H^k(\mathbb{R}^d))$ using the Fourier transform; the authors use this approach to prove some embedding theorems. This pivotal chapter concludes with the following complements:

(a) a discussion of periodic Sobolev spaces (the basic tool now is *Fourier series*);

(b) an application of spherical polynomials and spherical harmonics to the solution of the three-dimensional Laplace equation and other approximation problems;

(c) various Green's formulae.

The remaining chapters (Chapters 7 to 11) are more applied and computational than the previous ones. They form indeed the *theoretical numerical analysis* part of this book.

Chapter 7 is concerned with the "Variational Formulation of Elliptic Boundary Value Problems." Taking the classical homogeneous Poisson problem

$$\begin{cases} -\Delta u = f \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega \end{cases}$$

as a model, it is shown how to use Green's formula to derive the variational formulation of an elliptic linear partial differential equation. This leads to the Lax–Milgram lemma, a most important tool to show the existence and uniqueness of a solution to quite a broad class of linear variational problems in Hilbert spaces. Several applications are given; they include the Poisson equation with nonhomogeneous Dirichlet boundary conditions, the Neumann problem for the Helmholtz operator $-\Delta + I$ and for the Laplace operator $-\Delta$, the Helmholtz equation with mixed (Dirichlet–Neumann) boundary conditions, and the Neumann–Dirichlet problem for general second-order elliptic operators, i.e., problems of the form

$$\begin{cases} -\boldsymbol{\nabla} \cdot (\mathbf{A}\nabla u) + \mathbf{b} \cdot \boldsymbol{\nabla} u + cu = f \text{ in } \Omega, \\ u = 0 \text{ on } \Gamma_D, \\ \mathbf{A}\boldsymbol{\nabla} u \cdot \mathbf{n} = g \text{ on } \Gamma_N, \end{cases}$$

where $\mathbf{n}$ denotes the outward unit normal vector at the boundary Γ $(= \partial\Omega)$ of Ω (here $\mathring{\Gamma}_N \cap \mathring{\Gamma}_D = \emptyset$, $\overline{\Gamma}_N \cup \overline{\Gamma}_D = \Gamma$). Linear elliptic problems with the Robin boundary condition and the homogeneous Dirichlet problem for the biharmonic operator, i.e.,

$$\begin{cases} \Delta^2 u = f \text{ in } \Omega, \\ u = \dfrac{\partial u}{\partial n} = 0 \text{ on } \Gamma, \end{cases}$$

are discussed via exercises. The variational treatment of the linear elasticity equations is also considered in this chapter, which contains a discussion of mixed and dual formulations of some linear elliptic problems, including the celebrated Stokes problem

$$\begin{cases} -\Delta \mathbf{u} + \boldsymbol{\nabla} p = f \text{ in } \Omega, \\ \boldsymbol{\nabla} \cdot \mathbf{u} = 0 \text{ in } \Omega, \\ \mathbf{u} = \mathbf{0} \text{ on } \Gamma. \end{cases}$$

This important chapter concludes with the two following topics:

(i) A *generalized Lax–Milgram lemma* concerning the solution of linear variational problems such as

$$\begin{cases} u \in U, \\ a(u,v) = L(v) \quad \forall v \in V, \end{cases}$$

where U and V are two Hilbert spaces, not necessarily identical.

(ii) A brief discussion concerning the solvability (via the direct methods of the *calculus of variations*) of nonlinear Dirichlet problems such as

$$\begin{cases} -\boldsymbol{\nabla} \cdot (\alpha(|\boldsymbol{\nabla} u|)\boldsymbol{\nabla} u) = f \text{ in } \Omega, \\ u = 0 \text{ on } \Gamma. \end{cases}$$

Chapter 8 addresses the solution of linear variational problems by various types of Galerkin methods (the standard Galerkin methods, the Petrov–Galerkin method, and the generalized Galerkin method). Various convergence results and error estimates are derived, using, among other tools, Cea's lemma (incidentally, there is a typo on page 276, line 9, where the reference should be to section 7.7 and not 6.5).

Chapter 9 is a (nice) introduction to "Finite Element Methods" and their analysis. The reader is introduced to the subject via the following one-dimensional test problem:

$$\text{(BVP)} \qquad \begin{cases} -u'' + u = f \text{ in } (0,1), \\ u(0) = 0, \ u'(1) = b. \end{cases}$$

The authors discuss the finite element solution of (BVP) using piecewise linear and piecewise quadratic approximations. Many details concerning the construction and the solution of the resulting linear systems are given. After this introduction, the authors move to the solution of multidimensional problems (essentially two-dimensional) by finite element methods. The following concepts are discussed: finite element meshes made of triangles or quadrilaterals, reference elements, polynomial spaces on the reference elements, finite element interpolations (local and global), convergence and error estimates of the finite element approximation of second order linear elliptic problems (such as Poisson and Dirichlet's), and the condition number of the resulting linear systems.

Chapter 10 is a concise, but fairly complete, introduction to "Elliptic Variational Inequalities" and their numerical solution. Starting from some classical examples from mechanics such as the obstacle problem for an elastic membrane and the Signorini problem for an elastic body, the authors show how to obtain variational formulations generalizing the linear variational equations whose solution has been discussed in previous chapters. Roughly speaking, the resulting (variational) inequalities can be classified as elliptic variational inequalities of the first and second kinds. Next, the authors address the finite element approximation of these inequalities and the solution of the resulting finite-dimensional systems of equations and inequalities by methods based on regularization, Lagrange multipliers, etc. Uzawa-type algorithms are briefly mentioned on the occasion of the solution of a simplified friction problem.

Chapter 11 concerns the numerical solution of Fredholm integral equations of the second kind. It is the largest chapter of the book, and as such contains a detailed discussion of the many topics involved when solving such functional equations. These topics include projection methods via collacation techniques and Galerkin approximations, piecewise polynomial and trigonometric approximations, convergence results and error estimates for the above approximations, iterated projection methods, the Nyström method, collectively compact operator approximations, and the solution of nonlinear integral equations. The theoretical developments are illustrated by examples and numerical experiments.

The book concludes with Chapter 12, a chapter dedicated to the solution of some linear elliptic boundary value problems using boundary integral equations. The presentation is quite classical and covers the usual topics, namely, the boundary integral formulations of the interior and exterior Dirichlet and Neumann problems for the Laplace operator, single and double layer potentials, and boundary-integral equations of the second kind. After all of these preliminaries, the authors address the solution of boundary integral equations of the second kind by the Nyström methods discussed in Chapter 11; their discussion includes the results of numerical experiments showing how the approximation error behaves as a function of discretization parameters. The chapter concludes with the Galerkin solution of boundary integral equations of the first kind.

Overall, the book is clearly written, quite pleasant to read, and contains a lot of important material; and the authors have done an excellent job at balancing theoretical developments, interesting examples and exercises, numerical experiments, and bibliographical references. I sincerely regret that such a text did not exist when I was a student trying to apply variational methods (learned from J. L. Lions) to the solution of real-life problems from science and engineering. I wish that the graduate students I am trying to educate knew the contents of the present text, since it provides an excellent foundation for further and more specialized education and investigations in computational and applied mathematics.

Two minor criticisms:

- I regret that the authors did not give a direct proof of Theorem 3.3.3 concerning the projection on closed convex sets in Hilbert spaces. I think that the proof given in the book (based on general results on the optimization of convex functionals on convex sets in Banach spaces) is misleading.
- I found the discussion on the conjugate gradient methods (section 4.6) very interesting. However, I would have preferred that instead of problem (4.6.1) ($Au = f$), the authors had considered the following linear variational problem:

$$\text{(LVP)} \qquad \begin{cases} u \in V, \\ a(u,v) = L(v); \end{cases}$$

  problem (4.6.1) is a particular case of (LVP). Indeed, in many applications neither A nor f are known (we know their existence from the Riesz representation theorem, but constructing them from $a(\cdot,\cdot)$ and $L(\cdot)$ is more complicated, in general, than solving (LVP)). It can be easily shown that to solve (LVP) by a conjugate gradient algorithm, what is needed is the possibility of computing $a(v,w)$ and $a(v,w) - L(v)\, \forall v, w \in V$.

Reading and reviewing this book has been a most pleasant experience; I strongly recommend this text to colleagues and students.

REFERENCES

[1] R. Glowinski, *Finite Element Methods for Incompressible Viscous Flow*, Handbook of Numerical Analysis, to appear.

[2] P. G. Ciarlet, *Mathematical Elasticity, Volume* 1: *Three-Dimensional Elasticity*, North-Holland, Amsterdam, 1988.

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Asymptotic Modelling of Fluid Flow Phenomena. *By Radyadour Kh. Zeytounian.* Kluwer Academic, Dordrecht, The Netherlands, 2002. $161.00. xviii+545 pp., hardcover. ISBN 1-4020-0432-X.

Applied mathematicians have always found fluid mechanics to be a rich and interesting field because the basic equations (i.e., the Navier–Stokes equations) have an almost unlimited capacity for producing complex solutions that exhibit unbelievably interesting properties and because the dimensionless form of these equations contains a parameter (called the Reynolds number) which is usually quite large in technologically and geophysically interesting flows. This means that asymptotic methods can be used to obtain approximate solutions to some very interesting and important flow problems. These solutions usually turn out to be of nonuniform validity (i.e., they break down in certain regions of the flow), and matched asymptotic expansions have to be used to construct physically meaningful results.

However, advances in computer technology have led to the development of increasingly accurate numerical solutions and have thereby diminished the interest in approximate analytical results. But real flows (especially those that are of geophysical or engineering interest) are extremely complex and exhibit an enormous range of length and time scales whose resolution will probably remain well beyond the capabilities of any computer that is likely to become available in the foreseeable future. So simplification and modeling are still necessary, not only to meet the engineer's requirement for generating numbers but also for developing conceptual models that are simple enough to be