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Atkinson, Kendall(1-IA); Han, Weimin(1-IA)
Theoretical numerical analysis. (English. English summary)
A functional analysis framework. Texts in Applied Mathematics, 39.
Springer-Verlag, New York, 2001. xvi+450 pp. \$59.95. ISBN 0-387-95142-3
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The title of this book accurately describes its contents. The book covers the basic results of functional analysis that are needed in theoretical numerical analysis. The aim of theoretical numerical analysis is to provide rigorous existence and convergence proofs as well as error bounds for numerical algorithms. This study is called theoretical because the effect of roundoff error due to floating point computations is ignored; also the bounds obtained are often not very useful in practice because they are either hard to compute in practice or too pessimistic. Theoretical numerical analysis therefore is a rather advanced subject that requires considerable mathematical maturity.

The book starts with two chapters that introduce the reader to functional analysis. In these 89 pages the functional-analytic framework for dealing with theoretical numerical analysis problems is developed from scratch, starting with the definition of a linear space. The authors achieve this remarkable feat by avoiding a formalized framework for Lebesgue measure, integration, and distribution theory. Instead they use more standard results on the completion of normed spaces and the unique extension of densely defined bounded linear operators. This is a very interesting approach. Actually, these two chapters might form the basis for a functional analysis course in its own right.

The rest of the book covers a wealth of topics and might serve as the basis for several graduate courses. We briefly describe the chapters to give a rough idea.

Chapter 3 (Approximation theory) and 4 (Nonlinear equations and their solution by iteration) deal with the traditional topics in these fields from an abstract point of view. For example, existence and uniqueness of best approximations are studied, uniform error bounds, Fréchet and Gateaux derivatives are defined and the convergence of Newton's method in Banach spaces is proved.

Chapters 5--9 deal with partial differential equations and boundary value problems. Chapter 5 introduces the finite difference method. Chapter 6 discusses Sobolev spaces. In Chapter 7 the variational formulation of elliptic boundary value problems is given. The Galerkin method finds its place in Chapter 8 and finite element analysis is dealt with in Chapter 9.

The last three chapters deal with variational inequalities and integral equations. Among other things, this includes the approximation of elliptic variational inequalities, projection methods for integral equations and a homotopy argument for nonlinear problems.

The book ends with an extensive list of references and a good index.

The book is well written and accessible to students who have had the usual mathematics undergraduate courses in analysis, linear algebra and numerical analysis. It can be recommended as a textbook for several graduate mathematical courses and also as a reference work for numerical analysts who are interested in the general mathematical aspects of their work.

Reviewed by W. Govaerts

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