CS:5340 Homework 1

Due: Thu, 8/31

Notes: (a) Any problem numbers mentioned in the handout refer to problems in the textbook, by Arora and Barak. (b) It is possible that solutions to some of these problems are available to you via other theory of computation books or on-line lecture notes, etc. If you use any such sources, please acknowledge these in your homework *and* present your solutions in your own words. You will benefit most from the homework, if you seriously attempt each problem on your own first, before seeking other sources. (c) As mentioned in the syllabus, it is okay to form groups of two in solving and submitting homework solutions. But, my advice from (b) still applies: you will benefit most from the homework, if you seriously attempt each problem on your own first, before seeking help from your group partner. (d) Discussing these problems with any of your classmates is okay, provided you and your classmates are not being too specific about solutions. In any case, make sure that you take no written material away from these discussions *and* (as in (b)) you present your solutions in your own words. When discussing homework with classmates please be aware of guidelines on "Academic Dishonesty" as mentioned in the course syllabus.

1. For any binary string $x \in \{0,1\}^+$, let $dec(x)$ denote the non-negative decimal integer equivalent of x. Thus $dec(\texttt{000101}) = 5$. For $x, y \in \{0,1\}^+$ $x \leq y$ iff $dec(x) \leq dec(y)$.

   Pick an appropriate k and design a k-tape Turing machine M that computes the function $f : \{0,1\}^+ \times \{0,1\}^+ \to \{0,1\}$ where $f(x,y) = 1$ if $x \leq y$ and $f(x,y) = 0$ otherwise. M should run in $O(|x| + |y|)$ time on input (x, y).

   While it is tedious to specify "low level" details of Turing machines, it is even more tedious to read a "low level" description of a Turing machine. So in the interests of clarity and readability, you should use Example 1.1 from the textbook to model your solution. Make sure you clearly specify the number of tapes being used and how the input appears on the input tape.

2. Problem 1.5 (Chapter 1, Page 34). As the hint in the textbook suggests, the solution to this problem is obtained by modifying the "simulation" in the proof of Claim 1.6. You do not have to provide as much detail as I did in class; you can mimic the level of detail in the "proof sketch" for the claim in the textbook (Pages 17-18).

3. Solve Problem 1.10 (Chapter 1, Page 35). Advice on level of detail from Problem 1 applies.
