Lecture Notes: Social Networks: Models, Algorithms, and Applications
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1 Disease erradicability under voluntary vaccination

We previously talked the paper of Aspnes et al[1]. Their problem was studied in a pretty algorithmic way; they showed that a particular Nash equilibrium was particularly easy to characterize and find, but the social welfare optimum solution was hard to obtain (NP-hardness).

We now study a different paper, written by Perisic and Bauch [2], which uses a different approach to study games: simulation. The game is also different, in that players' decisions do not alter the payoffs of others.

The motivation behind this paper was that, if individuals behave rationally in voluntary vaccination settings, diseases may never be erradicated; if disease incidence is too small, individuals may choose not to vaccinate, and then keep the disease in small numbers, for example. (But we should note that smallpox was erradicated and polio is disappearing, both under voluntary vaccination.)

Let us take a look at Fig. 1. It is centered around two vertices: u and v. Vertex u, currently susceptible, is surrounded mainly by susceptible vertices, but it is connected by an edge to vertex v, which is currently infected. Edge $\{u, v\}$ has weight β, which represents the probability vertex v infects u.

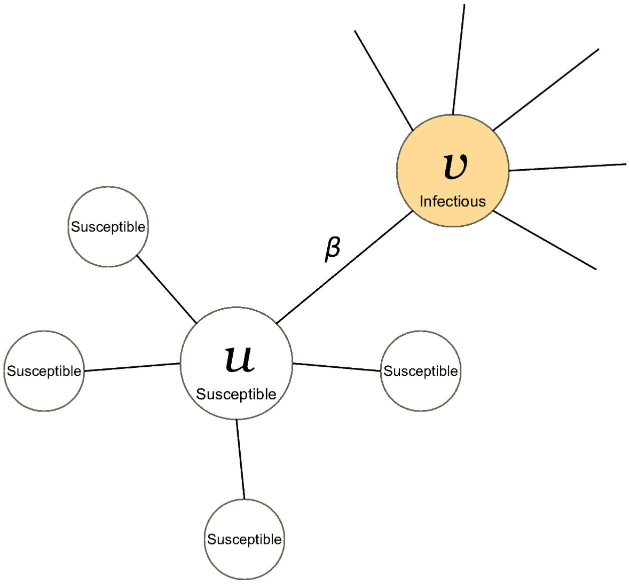

Figure 1: Susceptible u connected to infectious v.

Now, observe that vertex u may choose not to get vaccinated if β is small. But, if vertex v is surrounded by susceptibles, it will spread the disease with probability $1 - (1 - \beta)^{\deg(v)}$, which can become nonnegligible if $\deg(v) \gg 0$, even if $\beta \approx 0$. In this scenario, it may happen that the neighbors of u do not get vaccinated, the disease spreads from v and reaches them, u gets infected, and the disease keeps spreading further.

1.1 Population model

People are represented by a contact network $G = (V, E)$ modeled (and sampled) as an Erdös-Renyi random graph, with size $N = |V| = 5,000$ and average degree $\langle deg \rangle = c$. Note that G is static during each simulation run.

1.2 Infection model

Initially, I_0 individuals are chosen uniformly at random to be infected. Then, the disease spreads following a SEIR stages: susceptible-exposed-infectious-recovered, with the possibility of dying.

Recall that SEIR operates as follows: a person is first susceptible. Then, she/he gets infected and becomes *exposed*, i.e. infected but asymptomatic. After some time, she/he becomes infectious, i.e. starts transmitting the disease to her/his contacts. Finally, the person recovers after some time, acquiring *life long* immunity to the disease. (In practical terms, she/he cannot get infected again in the model.) However, Perisic and Bauch introduce the possibility of dying in this model; there is a probability d_{inf} that the person dies after the infectious stage instead a recovering (and surviving).

Parameter β represents the transmissibility of the disease, per day. (Transmissibility is the probability an infectious person infects another.)

1.3 Vaccination model

Each individual chooses to get vaccinated or not, during each day. (This is a discrete time model, where time is measured in days.)

Vaccines may, however, fail at immunizing susceptible individuals. A vaccine is effective with probability ϵ. Additionally, vaccination may kill a person. This happens with probability d_{vax}.

1.4 Behavioral model

In this model, people do not have complete knowledge about the characteristics and prevalence of the disease. Instead, they have some perceptions only. They do not know the actual transmissibility β, but rather they perceive β_{per}. Also, they do not know how many neighbors have been infected so far. They only know the number of infectious neighbors. Exposed people are indistinguishable from susceptibles.

Payoffs consist in the expected *life years*, i.e. the remaining life time, of the individual. Let us calculate the payoffs p_N of a susceptible that decides not to vaccinate. The perceived probability that individual gets infected is

$$\lambda_{per} = 1 - (1 - \beta_{per})^{n_{inf}},$$

where n_{inf} is the number of infectious neighbors of the individual. If an individual gets infected today, she/he may die (live 0 years) with probability d_{inf} or have a life expectancy of L years. And if escapes infection today, the individual expects to live α more years. Constant α absorbs all the consequences of all future decisions of the individual. Considering all this, we can obtain the payoff of a person who does not vaccinate:

$$p_N = \underbrace{\lambda_{per}(1 - d_{inf})L}_{getting\ infected\ today} + \underbrace{(1 - \lambda_{per})\alpha}_{the\ future}.$$

Note that $\alpha \leq L$ because of the acquired immunity considered in L.

The payoff p_V of a susceptible who decides to vaccinate is analogous to p_N. First of all, a person may die due to the vaccine with probability d_{vax}. Then, the vaccine may be effective with probability ϵ, leading to a life time of L. But if not effective, the person has a life expectancy of p_N (which considers the possibility of getting vaccinated in the future).

Finally, a person decides to get vaccinated today if $p_V > p_N$.

1.5 Concluding remarks

The authors wanted to see with this model how the disease unfolds and observe the behavior of people against a better vaccination model (socially optimal strategy).

Of their findings, they found that, if β_{per} is high, voluntary vaccination is possible; that vaccination scares (which consist in greater d_{vax} may have an important effect on incidence; that if the minimum degree increases, then diseases linger longer; and more.

Criticism of this work includes:

1. The population model may not be realistic; Erdös-Renyi random graphs do not capture many of the properties of real world networks, such as their degree distributions, clustering coefficient, assortativity, community structure, etc.
2. No aspects of one's payoff is a function of others' actions. For example, if I believe that the number of vaccinated people is large, then I should expect that I am less likely to contract the disease, thus making α closer to L.
3. Again on α, we know that it "absorbs" the expected outcome of all future decisions, yet, by making it constant, the opportunity of seeing how it varies over time is missed. Dynamic programming could have been used to approach this, for example.

2 Information networks

We now start change the topic from vaccination games to information networks. Since this topic is normally about the Web, we will quickly review how it developed.

2.1 History of the Web

Vannevar Bush (1890 - 1974), the founder of Raytheon was an American engineer, science administrator and Dean of the MIT school of Engineering, was known for his work on analog computing. He wrote ann interesting article in the Atlantic Monthly in 1945 about idea of "Memex", an adjustable microfilm viewer analogous to the structure of the World Wide Web, that would have served to store all the knowledge of an organization or society. The information for this machine would be *crowdsourced.*

Ted Nelson, greatly influenced by Vannevar Bush, introduced the term "hypertext". (He is the founder of `xanadu.com` circa 1960.)

In 1980, at the CERN, Tim Barners Lee suggested the ENQUIRE project, which was a wiki-like, hypertext-like organizational documentation engine.

In 1987, HyperCard, an application program for Apple Computer, Inc. combined both user-facing and developer-facing in a single application which allows rapid turnaround and immediate prototyping, allowing users to author custom solutions to problems with their own personalized interface.

In 1988, Wide Area Information Servers (WALS) developed as a project of Thinking Machines corporation that started in Boston which a client-server text searching system that uses the ANSI standard Information Retrieval Service Definition and Protocol Specifications for Library Applications to search inmdex databases on remote computers. This corporation provided a service called the Directory of Servers which is like a WALS server like any other information source but contained information about the other WALS servers on the internet.

In 1989, World Wide Web (WWW) is a system of interlinked hypertext document accesses via the internet was founded by Tim Berners Lee using concepts from his hypertext systems like ENQUIRE etc.

In 1990 we have Archie, an index of FTP (File Transfer Protocol) sites that downloaded the index files of servers and used GREP for search. (GREP is a tool for searching/parsing text files making use of regular expressions.)

In 1991 we have Gopher (from the University of Minessota), which was a menu text-based engine for hierarchical FTP sites.

In 1992, we have Veronica, an index of Gopher servers/sites.

The WWW appeared in the late 1990s. Web pages were static, so the Web was really a point and click fancy version of FTP with text. These web pages contained hyperlinks, and markups for presentation. After that, images were incorporated and later web pages became dynamic. First, pages were generated from database queries and later using CGI. Then, dynamism on the client appeared (form validation, Javascript). Finally, the Web became bidirectional in that not only servers provided the information, but also the users.

Despite all this accelerated evolution, the Web is still a directed graph. The Web has many connected components. Of importance are algorithms for finding the Strongly Connected Components. They are normaly depth-first, like Tarjan's [3].

References

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[2] A. Perisic, C.T. Bauch (2009). Social contact networks and disease eradicability under voluntary vaccination. PLoS Computational Biology, 5(2):e1000280.

[3] R.E. Tarjan (1972). Depth-first search and linear graph algorithms. SIAM Journal on Computing 1 (2): 146160.