Lecture Notes: Social Networks: Models, Algorithms, and Applications
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1 Game-theoretic Modeling of Traffic Congestion

Vaccination (Inoculation) games[1]: Let us consider the following: V = (0, 1, 2,..., n-1) is a set of players. G=(V,E) is the contact network. Each player i makes a strategy choice a_i=[0,1]. Interpret a_i as the probability that i chooses to get inoculated. Let $\hat{a}$ = (a_0, a_1, ..., a_{n-1})∈[0,1]^n denote the strategy choice of all players.

Strategy choice of all individuals
Let $G_{\hat{a}}$ denote the random subgraph of G in which a node exists with probability 1-a_i.

Attack Model: An attacker adversly chooses a node i∈V uniformly at random and disease spreads deterministically through out the conected component in $G_{\hat{a}}$ containing i.

Cost for Player i:

$$Cost(i) = a_i(C) + (1 - a_i) * L * P_i(\hat{a})$$

Example:
Pure Strategy Choice = (0,0,1,1,0,0)
E=(0,1), (0,3), (1,2), (3,2), (2,4), (2,5)
Costs
0⇒ $L * 1/3$
1⇒ $L * 1/3$
2⇒ C
3⇒ C
4⇒ $L * 1/3$
5⇒ $L * 1/3$

Social Cost:

$$\sum_{i \in V} cost(i)$$

For a pure strategy choice $\hat{a}$ ∈0,1^n, the social cost is:

$$C * (of individuals for which a_i = 1) + L/n \sum_{j=1}^{t} k_j^2$$

There are t connected components in $G_{\hat{a}}$, with component j having size k_j.

2 Characterization of Nash Equilibrium

Theorem 1 *A strategy choice $\hat{a} = (a_0, a_1, \ldots, a_{n-1})$ is a Nash Equilibrium, iff:*

$\forall i$ such that a_1 = 1, $P_i(\hat{a}) \geq c/L$
$\forall i$ such that a_1 = 0, $P_i(\hat{a}) \leq c/L$
$\forall i$ such that 0<a_1<1, $P_i(\hat{a}) = c/L$
every connected component in $G_{\hat{a}}$ has size at most Cn/L
if we add any node i to $G_{\hat{a}}$, the connected component containing i has size at least Cn/L

Consider player $i \in V$ with $a_i = 1 \Rightarrow cost(\text{i}) = C$.

Consider what happens if i chooses $a_i = 0$.

Then cost(i) = L$P_i(\hat{a})$

L$P_i(\hat{a}) \geq C \Rightarrow$P$_i(\hat{a}) \geq c/L$ unvaccinated components vs. vaccinated nodes: C(i) is the connected component of player i who chooses not to get vaccinated.

$$L * |C(i)|/n \geq C \Rightarrow |C(i)| \geq Cn/L$$

Example:
Suppose C/L = 1/2, let E=$(0,1),(0,3),(1,2),(3,2),(2,4),(2,5)$
NE: player 2 gets vaccinated

How do we compute a pure strategy Nash Equilibrium? Given G=(V,E), L and C.
-Initially everyone is vaccinated.
-Repeatedly unvaccinate individuals.

Recall that the social cost is:

$$C * (of individuals for which a_i = 1) + L/n \sum_{j=1}^{t} k_j^2$$

(1) NP-complete
(2) Also, hard to approximate
(3)There are $O(log^2 n)$ approximations using multicommodity flow algorithms

Price of Anarchy: Consider a star graph of n nodes with node 0 at the center
-$C/L = (n-1)/n \Rightarrow Cn/L = n-1$
-NE: any of the nodes gets vaccinated, $\Rightarrow C + L(n-1)^2/n \approx C + Ln$
-Social Optimum: the center node gets vaccinated $C + L(n-1)/n \approx C + L$

James Aspnes, Kevin Chang and Aleksandr Yampolskiy. Inoculation strategies for victims of viruses ans the sum-of-squares partition problem. *Journal of Computer and System Sciences, 72(6):1077-1093, September 2006.*