Lecture Notes: Social Networks: Models, Algorithms, and Applications
Lecture 1: April 10, 2012
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1 Reminders

The final presentation will on May 8th from 5:30-10pm in 210 MLH. Presentations are to be a conference style talk of about 10 minutes in length. NB–conference style means come with a PowerPoint presentation!
The second draft of the paper is due this Sunday (4/15) by midnight. Remember to turn it in to both Professors Pemmaraju and Segre in pdf format.
Reviewers will be assigned Thursday, 5/12.

2 Review: Basic Game Theory

3 Client Game Recall the 3-client game from last class. Given the payoff matrix in table 1,

Table 1: Two Businesses Compete for Three Clients

		Firm 2		
		A	B	C
Firm 1	A	4,4	0,2	0,2
	B	0,0	1,1	0,2
	C	0,0	0,2	1,1

we can derive the best responses shown in table 2. Note that there is no dominating strategy. However, we can see that given any choice except (A, A) the players will deviate. An example deviation would be the choice (B, C). In that case Firm 1 will choose to move to C. In fact, (A, A) is the unique pure Nash equilibrium in this game.

Table 2: Best Response to Payoff Matrix in Table 1

$BR_1(A) = A$	$BR_2(A) = A$
$BR_1(B) = B$	$BR_2(B) = C$
$BR_1(C) = C$	$BR_2(C) = B$

Definition 1 Nash Equilibrium: *A strategy choice* (S, T) *is a Nash equilibrium if* $BR_1(T) = S$ *and* $BR_2(S) = T$. *Best responses can also be sets of responses.*

Nash Equilibria can be unsatisfying. Although they are a convenient way to express properties of a payoff matrix, pure Nash equilibria also produce the following behavior.

Multiple Nash equilibria are possible as shown in table 3, where the Nash equilibria are highlighted in bold.

Table 3: Presentation Game

		Partner 2	
		PowerPoint	Keynote
You	PowerPoint	**1,1**	0,0
	Keynote	0,0	**2,2**

Table 4: Attack-Defend Game

		Defender	
		H	T
Attacker	H	-1,1	1,-1
	T	1,-1	-1,1

There may be no pure Nash equilibrium as shown in table 4. In fact this is a feature of most *zero-sum* games, where the sum of the payoffs are zero.

However, all games do have a *mixed* Nash equilibrium strategy.

3 Mixed Strategies

A mixed strategy is a probability distribution over the strategy set of a player. In games that allow mixed strategies, each player is choosing a probability distribution.

3.1 Matching Pennies

In the matching pennies game, the payoffs are like those in table 4. $((p, 1-p), (q, 1-q))$ is a strategy choice, where p, q are the probability of one player picking heads.

3.1.1 Expected Payoffs

In the case of mixed strategies, players use expected payoffs to decide which strategies to pick. In the case of the Matching Pennies game, P1 expects:

$$-pq + p(1-q) + (1-p)(q) + -(1-p)(1-q) = -4pq + 2(p+q) - 1$$

. P1 uses this expression to determine its best response by solving

$$max_{p\in[0,1]}(-4pq^* + 2(p+q^*) - 1)$$

for fixed $q = q^*$.

Now think of this equation as a line through the space of strategies. Since it is a line the maximum will either be on one side ($p = 0$ or $p = 1$) or the line will be horizontal and there will be no maximum. Consider three cases:

1. $q* < .5$ implies that the slope is positive, and therefore optimal $p = 1$.

2. $q* > .5$ implies that the slope is negative, and therefore optimal $p = 0$

3. $q* = .5$ implies that slope is 0, and therefore optimal $p = .5$

This tells us that the choice of $p \neq .5$ or $q \neq .5$ is not a Nash equilibrium, and that $p = .5, q = .5$ is a Nash equilibrium. This shows that matching pennies has a unique Nash equilibrium at $((1/2, 1/2), (1/2, 1/2))$

We can generalize this result according to the following theorem.

Theorem 2 *Every finite game has a Nash equilibrium (possibly by using mixed strategies).*

Additionally we will introduce two definitions.

Definition 3 *Pareto optimality: A strategy choice (S, T) is Pareto optimal if there is no other strategy choice that gives each player at least as much payoff and some player strictly greater payoff.*

Definition 4 *Social welfare maximization: A strategy choice (S, T) is said to maximize social welfare if the sum of the payoffs for this choice is maximal. Functions other than sum can also be used to calculate maximization of social welfare.*

Traffic Modeling using Game Theoretic Notions Model: Suppose $G = (V, E)$ is a directed graph with special nodes s and t which operate as a source of traffic and the destination (or target) of traffic respectively. Each edge in E is labeled with function $T_e(x)$ that denotes travel time along edge e as a function of the volume of traffic x on e.

Each player picks an st-path in G

The payoff of a player is the negative of the travel time of that player.

Questions:

1. Are there dominant strategies?

2. Are there pure Nash equilibrium strategies?

3. If so, how far are these from being welfare maximizing?