Lecture Notes: Social Networks: Models, Algorithms, and Applications
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1 Introduction to Game Theory

Game Theory is the study of strategic decision making and has a long background in economics, psychology and various other domains. Games are used to model situations in which there is interaction between decision makers and the payoff that a decision maker realizes depends not just on his/her decision, but also on the decisions made by others.

Some example applications of Game Theory include:

1. **Games on Networks** *Distributed algorithms must decide how much bandwidth to provide to networks.*
2. **Vaccination Games** *Games where we consider the incentives associated with decision-making regarding vaccination policies.*
3. **Congestion Games** *A game where we define players and resources and the payoff is determined by the resources a player chooses, as well as the number of players choosing those same resources.* [2, 5]

2 Games with Strictly Dominate Strategies

2.1 Examples: (Easly-Kleinberg Chapter 6)

2.1.1 Exam vs. Presentation Game

The school semester is drawing to a close and one class requires that you must prepare for both an exam and a presentation. However, lacking time to sufficiently prepare for both, you must make a decision on how to best allocate your preparation time. While the exam is taken individually, the presentation is a group project with one partner from the same class. Based on the decision you make, assume the following grade outcomes:

Exam
If you study: Expected Score **92**
If you *don't* study: Expected Score **80**

Presentation
If both participate: **100**
If only one participates: **92**
If neither participates: **84**

Because your grade is also a function of the decision made by your group partner, in order to formulate the space of possible outcomes, we define a payoff matrix by enumerating the possible grade combinations in Table 2.1.1

	Exam	Presentation
Exam	(**88**, 88)	(**92**, 86)
Presentation	(**86**, 92)	(**90**, 90)

Table 1: Payoff matrix, where cell values correspond to (**your score**, your partner's score)

We want to analyze this game and determine the strategies the players will choose. Proceed by writing out our strategy:

1 Partner chooses exam - Your best response: **Exam**

2 Partner chooses presentation - Your best response: **Exam**

Observe that the best strategy, regardless of the choice made by your opponent, is to study for the exam.
$\Rightarrow$ E is the strictly dominate strategy for you because of symmetry
$\Rightarrow$ E is the strictly dominate strategy for your partner as well

Note that rationally acting agents (i.e., individuals always make choices that maximize their benefits while minimizing their costs) choose exam. We make several assumptions under this model:

1. Everything players care about is encoded in the payoff matrix
2. Players have complete knowledge and share common information. [1]

2.1.2 Prisoner's Delimma

Two individuals have been arrested in connection to a crime, but without a confession the police lack enough evidence to convict either. The police interrogate each person in separate rooms with the goal of getting each individual to testify against the other. Depending on how each individuals confess or refuse to confess, each is tried and sentenced to some number of years in jail, defined by the following payoff matrix:

	Confession	No Confession
Confession	(**4**, 4)	(**0**, 10)
No Confession	(**10**, 0)	(**1**, 1)

Table 2: Payoff matrix; units are years sentenced to jail

As in the previous example, there exists a strictly dominating strategy; both prisoner's have incentive to confess and testify against each other.

[1] For games of *incomplete* knowledge, refer to JC Harsanyi's work [1] for which he received the 1994 Nobel Prize in Economics

3 Nash Equilibrium

While the previous two examples had strictly dominate strategies, we can construct examples where no such strategy exists. Consider the following example:

For Firm 1, Firm 2 and Clients A, B, C

- Firm 1 is small and cannot do business on its own with a client.
- If Firm 1 does business with a client on its' own, its payoff is 0.
- If Firm 1 and Firm 2 both approach a client for business, each gets half the business.
- A is a big client and needs both firms. B and C are smaller and can work with just one firm.
- For clients B and C if Firm 2 approaches either by itself, it gets their complete business.
- Doing business with A is worth 8. Doing business with B,C is worth 2.

	A	B	C
A	4, 4	0, 2	0, 2
B	0, 0	1, 1	0, 2
C	0, 0	0, 2	1, 1

Player 1	Player 2
$bR_1(A) = A$	$bR_2(A) = A$
$bR_1(B) = B$	$bR_2(B) = C$
$bR_1(C) = C$	$bR_2(C) = B$

Table 3: Payoff matrix (top) and corresponding best response functions (bottom).

If we formulate a *Best Response Function* based on our opponents choices, we get the strategies outlined in the table above. Notice that there are no strictly dominate strategies. In this scenario, what might be the solution to the game? This question brings us to the idea of *Nash Equilibrium* [4, 3], which can be described in the following rather intuitive way:

Given a choice T by Player 2, Player 1 want to play his best response to T

Analyzing the previous example the, we find that the choice (A,A) resulting in outcome (4,4) is the unique Nash Equilibrium

References

[1] J.C. Harsanyi. Games with incomplete information played by" bayesian" players, i-iii. part i. the basic model. *Management science*, pages 159–182, 1967.

[2] R. Kleinberg, G. Piliouras, and É. Tardos. Multiplicative updates outperform generic no-regret learning in congestion games. In *Proceedings of the 41st annual ACM symposium on Theory of computing*, pages 533–542. ACM, 2009.

[3] J. Nash. Non-cooperative games. *The Annals of Mathematics*, 54(2):286–295, 1951.

[4] J.F. Nash et al. Equilibrium points in n-person games. *Proceedings of the national academy of sciences*, 36(1):48–49, 1950.

[5] A. Roth. The price of malice in linear congestion games. *Internet and Network Economics*, pages 118–125, 2008.