Efficiency of List Operations

MARCH 28

Some ways of modifying lists are faster than others…

- Consider this code snippet:

```
L = []
for i in range(100000):
        L.insert(0, i)
```

This constructs a list of one hundred thousand integers:
99999, 99998, 99997, … , 3, 2, 1, 0.

How does this compare in speed to the other ways one can do this in Python?

Other ways of doing the same thing…

```
L = []
for i in range(100000-1, 0, -1):
    L.append(i)
```

```
L = []
for i in range(100000):
    L = [i] + L
```

Here is a puzzle

- When I ran these different ways and measured the running time, here is what I got (in seconds):

  0.031, 5.063, 34.55.

Can you match the running times with the code snippets?

- The medium-speed code is more than 150 times slower than the fastest code. The slowest code is more than 1000 times slower than the fastest code!

Mental model of how lists are implemented

• Suppose we execute `L = [12, 15, 11, 4]`.

• A block of memory is allocated and the items 12, 15, 11, and 4 are stored consecutively at the beginning of this block.

• This allows efficient access to all elements of the list. The location of `L[i]` in memory is simply i + starting location of `L`.

• This guarantees that every element in the list, no matter what its index is, can be accessed equally quickly. This kind of access is called *random access*.

Consequences of this implementation

- `append` is fast. Consider `L.append(e)`. The length of `L` is known and hence the location of the first empty slot following `L` is also known. The element `e` is stored in this slot.
- Notice that the running time of the `append` operation is *independent* of the size of L. append takes the same amount of time, no matter how large L is.
- We say that the running time of append is *constant*. This does not mean that it is the same across different machines.

Consequences of this implementation

- `insert` and `remove` can be slow because these might cause a large portion of the list to "shift."
- For example, `L.insert(0, e)` causes every element in the list to move one slot. This creates a "hole" at the beginning of the list for element `e`.
- This also means that insert operations towards the end of the list are cheaper than those at the beginning of the list.
- In the *worst case* insert takes time that is proportional to the length of L.
- In other words, insert is said to take *linear* time in the worst case.

Analyzing the code snippets

```
L = []
for i in range(n-1, 0, -1):
    L.append(i)
```

- Assume that append takes time c, a constant that has nothing to do with n.
- Since the for-loop executes n times, the running time of this code snippet is $c\,n$.
- Since c is a constant this is a linear function in n.

Analyzing the code snippets

```
L = []
for i in range(n):
    L.insert(0, i)
```

- After the for-loop has executed i times, we have a list of length i. We know that `insert` takes time $c\ i$ on this list.
- Therefore the total running time is

$$c\ (1 + 2 + 3 + \ldots + n\text{-}1) = c\ n\ (n + 1)/2.$$

- Since c is a constant this is a quadratic function in n.