22C:153 Self-Evaluation Exam

Solutions

(1) $\lg(n!) = \Theta(n \log n)$ – True (Using Sterling's approximation).

(2) $F_n = O((1+\sqrt{5})^n/2^n)$ – True.

(3) $2^{\lg n} = \Omega(n)$ – True.

(4) $T(n) = T(n-1) + 1/n \rightarrow$ This is a harmonic series and so is $\Theta(\ln n)$.

(5) $T(n) = T(n-1) + \log n \rightarrow \Theta(n \log n)$.

(6) $T(n) = 7 \cdot T(n/2) + n^2 \rightarrow \Theta(n^{\lg 7})$.

(7) When the two lines are added, `PARTITION` returns a balanced partition even in the worst case. So, the running time is $O(n \lg n)$.

(8) $O(n \lg n + i)$.

(9) $O(n + i \lg n)$. We can build a binary heap in $O(n)$ and each `EXTRACT-MAX` takes $O(\lg n)$.

(10) $O(n + i \lg i)$

(11) Consider the following activities $[3,6], [0,4], [5,9]$ with the first element representing the starting time and the second element representing the finish time for each activity. $[3,6]$ will be picked preventing the choice of $[0,4]$ or $[5,9]$. Optimal solution is {[0,4],[5,9]}.

(12) Let $A = B \cdot B^T = (a_{ij})$. Then

$$a_{ij} = \begin{cases} \text{Number of edges incident on } i & \text{if } i = j \\ -1 & \text{if there is an edge between } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$$

(13) The values of $d[v]$ and $f[v]$ are:

v	$d[v]$	$f[v]$
000	1	22
001	2	21
002	3	14
010	23	24
011	4	13
012	5	6
100	15	20
101	16	19
102	17	18
110	7	12
111	8	11
112	9	10

(14) Tree edges are:

$000 \to 001 \to 002 \to 011 \to 012$
$011 \to 110 \to 111 \to 112$
$001 \to 100 \to 101 \to 102$

Back edge is:

$012 \to 002$

Forward edges are:

$000 \to 101$ and $001 \to 012$

Cross edges are:

$010 \to 011$
$102 \to 111$
$010 \to 111$
$101 \to 112$

(17) Yes.

(18) No.

(19) The shortest paths from vertex 1 to other vertices are:

$1 \to 2$
$1 \to 2 \to 3$
$1 \to 4$
$1 \to 2 \to 5$
$1 \to 2 \to 5 \to 6$
$1 \to 4 \to 7$
$1 \to 2 \to 5 \to 8$
$1 \to 2 \to 5 \to 8 \to 9$

(20) Dijkstra's algorithm will not correctly compute shortest paths from vertex 1 for the following graph. Bellman-Ford will.