22C:153 Homework 2

Due: Thursday, 3/20

On dynamic programming:

(1) Problem 15.3-5 (page 350).

(2) Devise an algorithm that takes as input a convex polygon, described by a sequence of points $p_1, p_2, \ldots, p_n$ specified in counterclockwise order around the polygon, and produces as output a *triangulation* of minimum weight. A triangulation of a convex polygon is a maximal set of non-intersecting diagonals. The weight of a triangulation is the sum of the lengths of the line segments in it.
Hint: Using dynamic programming an $O(n^3)$ algorithm is possible.

(3) Problem 15-3 (page 364).

(4) The *subset-sum ratio problem* is the following.

> Given n positive integers $a_1 < a_2 < \ldots < a_n$, find two disjoint, non-empty subsets $S_1, S_2 \subseteq \{1, 2, \ldots, n\}$ with $\sum_{i \in S_1} a_i > \sum_{i \in S_2} a_i$, such that the ratio
>
> $$\frac{\sum_{i \in S_1} a_i}{\sum_{i \in S_2} a_i}$$
>
> is minimized.

Devise a pseudo-polynomial time algorithm for this problem.

On max-flow

(5) You are given a directed graph $G = (V, E)$ with distinguished vertices s and t. Devise an algorithm with running time $O(|V|^2)$ to compute a maximal set of shortest paths from s to t.

(6) Problem 26-2 (pages 692-693).

(7) Problem 26-4 (page 694).

(8) Problem 26-5 (page 694).

(9) Consider the following flow network with edge capacities 1, R, and M. Assume that $M \geq 4$ is an integer and $R = (\sqrt{5} - 1)/2$. We will show that there is an infinite sequence of augmentations possible for this flow network.

(a) Let $a_0 = 1$, $a_1 = R$, and $a_{n+2} = a_n - a_{n+1}$ for any $n \geq 0$. Show by induction that $a_n = R^n$.

(b) Start with an intial flow f that assigns 1 unit of flow to edges (s, c), (c, b), and (b, t) and 0 units everywhere else. Now notice that the residual capacities of edges (c, d) and (a, b), are a_0 and a_1 and the residual capacity of (b, c) is 1. Describe a sequence of 4 augmentations after which the residual capacities of edges (c, d), (a, b), and (b, c) are $a_2 = 1 - R$, $a_3 = 2R - 1$, and 1.

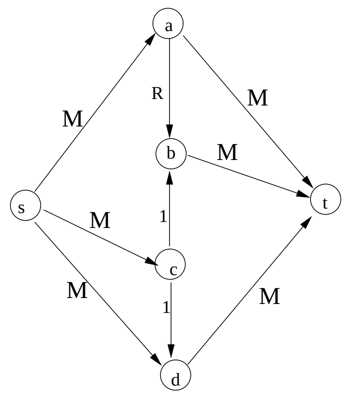

Figure 1: A flow-network on which the Ford-Fulkerson method does not terminate.

(c) Call the sequence of 4 augmentations described in (b) a *round*. Generalize your solution to (b) and show a sequence of n rounds after which the residual capacities of edges (c, d), (a, b), and (b, c) are respectively a_{2n}, a_{2n+1}, and 1. What is the value of the flow at this point? What is the limiting value of the flow as $n \to \infty$.

(10) Problem 26.2-9 (page 664).