22C:131 Homework 3

Due: Tuesday, 11/20

Notes: (a) Undergraduate students are required to solve the first 6 problems in the homework. Graduate students are required to solve all problems. The problem numbers, definition numbers, and Theorem numbers refer to the textbook, by Arora and Barak. (b) It is possible that solutions to some of these problems are available to you via other theory of computation books, on-line lecture notes, wikipedia, etc. If you use any such sources, please acknowledge these in your homework *and* present your solutions in your own words. You will benefit most from the homework, if you sincerely attempt each problem on your own first, before seeking other sources. (c) It is okay to discuss these problems with your classmates. Just make sure that you take no written material away from these discussions *and* (as in (b)) you present your solutions in your own words. When discussing homework with classmates please be aware of guidelines on "Academic Dishonesty" as mentioned in the course syllabus.

1. The paper "Robust Simulations and Significant Separations" by Lance Fortnow and Rahul Santhanam has a very nice, new and short proof of the non-deterministic hierarchy theorem. You will find the paper at `http://arxiv.org/abs/1012.2034` and you can also read a discussion of this proof in the blog called *Computational Complexity* (post on Tuesday, April 5 2011), one of whose authors is Lance Fortnow. You might find the paper hard to read because the new proof of the non-deterministic hierarchy theorem is just one small part of the paper. A clear presentation of the proof appears in the lecture notes of Jonathan Katz at the University of Maryland (`http://www.cs.umd.edu/~jkatz/complexity/f11/lecture4.pdf`, see Theorem 3.) To answer this question you will need to study the new proof of the non-deterministic hierarchy theorem from Prof. Katz's lecture notes.

   (a) Carefully show that M_L runs in $O(G(n))$ time.

   (b) Figure 3.1 in Arora-Barak is a pictorial description of Cook's proof of the non-deterministic hierarchy theorem. Draw a diagram that similarly describes the Fortnow-Santhanam proof. Clearly, there is no single answer to this problem. In designing your diagram, focus on clarity and expressiveness.

2. This problem is on the proof of Ladner's Theorem.

   (a) Describe (using some reasonable pseudocode) a polynomial-time algorithm for computing the function H.

   (b) Show that your algorithm indeed runs in polynomial time.

   (c) Let us generalize the definition of H by replacing the two occurrences of $\log \log n$ by an arbitrary function $f : \mathbf{N} \rightarrow \mathbf{N}$ and then replacing the occurrence of $\log n$ by an arbitrary function $g : \mathbf{N} \rightarrow \mathbf{N}$. Describe constraints we have to place on f and g for the proof (as described in Arora-Barak) to go through without any significant changes. In other words, view $f(n) = \log \log n$ and $g(n) = \log n$ as particular instances of functions f and g that makes the proof work and describe the family of functions f and g for which the proof will work.

3. Problem 4.1 on Universal TM for space-bounded computation and the space hierarchy theorem.

4. Show that TQBF restricted to formulas where the part following the quantifiers is in CNF is still PSPACE-complete.

5. Show that SPACE TMSAT (defined on Page 83) is PSPACE-complete.

6. In class we discussed two definitions of *logspace reductions*. The first definition appears as Definition 4.16 in Arora-Barak and the second definition (from Sipser's textbook) involves a TM with a write-only output tape. Show that the two definitions are equivalent. In other words, show that language A is logspace reducible to language B in the first sense iff language A is logspace reducible to language B in the second sense.

7. Show that 2SAT is NL-complete.

8. Problem 4.12 on the complexity class SC, which stand's for *Steve's Class*, named in honor of Steve Cook. Later in Chapter 6 we will study the class NC, which stands for Nick's Class, named in honor of Nick Pippenger who has done a lot of work on circuit complexity.
