Assignment 5: Turing machines and decidability (265 points)

November 3, 2009

This assignment requires you to solve the following problems:

1. (15 points) This exercise concerns TM M_2 whose description appears in Example (3.4, First edition) 3.7. In each of the following three parts, give the sequence of configurations that M_2 enters when started on the indicated string:
   (a) (5 points) 0
   (b) (5 points) 000
   (c) (5 points) 000000
2. (20 points) Explain why the following is not a description of a legitimate Turing machine:
   M_{bad}="The input is a polynomial P over variables x_1, ...,x_k.
   (a) Try all possible settings of x_1,...,x_k to integer values.
   (b) Evaluate P on all of these settings.
   (c) If any of these settings evaluates to 0 *accept*; otherwise *rejects*."
3. (40 points) Show that the collection of decidable languages is closed under the operations:
   (a) Concatenation (10 points)
   (b) Star (10 points)
   (c) Complementation (10 points)
   (d) Intersection (10 point)
4. (30 points) Show that the collection of Turing-recognizable languages is closed under the operations of:
   (a) Concatenation (10 points)
   (b) Star (10 points)
   (c) Intersection (10 points)

5. (20 points) Show that a language is decidable iff some enumerator enumerates the language in lexicographic order.

6. (20 points) Show that every infinite Turing-recognizable language has an infinite decidable subset.

7. (20 points) Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language (5 points) and show that this language is decidable (15 points).

8. (20 points) Consider the problem of determining whether a DFA A recognizes the language $L = \Sigma^*$ for Σ a finite alphabet. Express this problem as a language (5 points) and show that this language is decidable (15 points).

9. (10 points) Let $A_{\epsilon CFG} = \{\langle G \rangle | G \text{ is a CFG that generates } \epsilon\}$. Show that $A_{\epsilon CFG}$ is decidable.

10. (20 points) Let A and B be two disjoint languages. Say that a language C separates A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

11. (20 points) Let $S = \{\langle M \rangle | M \text{ is a DFA that accepts } w^{\mathcal{R}} \text{ whenever it accepts } w\}$. Show that S is decidable.

12. (20 points) Let A be a Turing-recognizable language consisting of all descriptions of Turing machines, $\{\langle M_1 \rangle, \langle M_2 \rangle, \ldots\}$ where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A.

Due date is Wednesday 18 November 2009.