# **Assignment 3** (240 points)

Please solve the following problems:

1. (20 points) Consider the function $f : \{0,1\}^* \to \{0,1\}^*$ defined by:

   - $f(\epsilon) = \epsilon$, $f(0) = 0$, $f(1) = 1$;
   - For $w_i \in \{0,1\}$ and $k \geq 2$, $f(w_1 w_2 \ldots w_k) = w_1 w_2 w_1 w_3 \ldots w_1 w_k w_1$.

   Determine whether or not $f$ is a DGSM function and prove it.

2. (20 points) Let $\Sigma = \{0,1\}$ be an alphabet. Give regular expressions that generate the following languages over $\Sigma$:
   $L_1 = \{w \| w| \leq 5\}$
   $L_2 = \{w | w$ starts with `0` & has odd length, or starts with `1` & has even length $\}$
   $L_3 = \{w | w$ does not contain the substring `110` $\}$

3. (20 points) In certain programming languages, comments appear between delimiters such as /∗ and ∗/ in C. Let $L$ be the language of all valid delimited comment strings. A member of $L$ must begin with /# and must end with #/ but have no intervening #/. For simplicity, we say that comments themselves are written with only the symbols `a, b`. Hence, the alphabet of $L$ is $\Sigma = \{\texttt{a}, \texttt{b}, /, \#\}$.

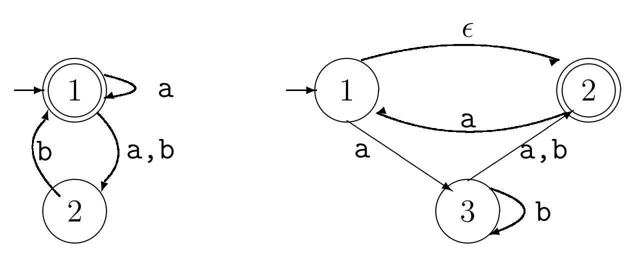
   (a) Give a DFA that recognizes $L$ (10 points).

   (b) Give a regular expression that generates $L$ (10 points).

4. (20 points) Let $F$ be the language of all strings over $\{0,1\}$ that do not contain a pair of `1`s that are separated by an odd number of symbols. Give the state diagram of the DFA with 5 states that recognize $F$.

   **Hint:** you may find it helpful first to find a 4-state NFA for the complement of $F$.

5. (20 points) Use the construction which show that every NFA has an equivalent DFA to convert the two NFAs whose transition diagrams are in the figure below in to equivalent DFA.



6. (20 points) Let $\Sigma_2 = \{\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\}$. Here $\Sigma_2$ contains all columns of 0s and 1s of height two. A string of symbols in $\Sigma_2$ gives two rows of 0s and 1s. Consider each row to be a binary number and let $C = \{w \in \Sigma_2^* |$ the bottom row of $w$ is three times the top wow $\}$. For example $\begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \in C$ but $\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin C$. Show that $C$ is regular.

   **Hint:** you may assume that for any language $A$ if $A$ is regular then the language $A^{\mathcal{R}} = \{w^{\mathcal{R}} | w \in A\}$ is also regular. Thus, you may design an automaton which read the input backwards. The symbols it reads are columns of the form $\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$, $d_1, d_2 \in \{0, 1\}$. The states must be such that if binary sum $d_1 + d_1 + d_1$ is not $d_2$ (having in view the possible overflow) than the automaton reject, otherwise it performs further the addition of the next digits.

7. (20 points) Let $\Sigma_2 = \{\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\}$. Here $\Sigma_2$ contains all columns of 0s and 1s of height two. A string of symbols in $\Sigma_2$ gives two rows of 0s and 1s. Consider the top and bottom rows to be strings of 0s and 1s and let $E = \{w \in \Sigma_2^* |$ the bottom row of $w$ is the reverse of top row of $w$ $\}$. Show that $E$ is not a regular language.

8. (20 points) Let $B_n = \{a^k |$ where $k$ is a multiple of $n$ $\}$. Show that for each $n \geq 1$, the language $B_n$ is regular.

9. (20 points) Let $C_n = \{x | x$ is a binary number that is a multiple of $n\}$. Show that for each $n \geq 1$ the language $C_n$ is regular.

10. (20 points) Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection, and complement.

    (a) $L_1 = \{0^n 1^m 0^n | m, n \geq 0\}$ (10 points).

    (b) $L_2 = \{w | w \in \{0, 1\}^*$ is not a palindrome $\}$ (10 points).
    **Note:** a palindrome is a string that reads the same forward and backward. For example `anna` is a palindrome.

    (c) $L_3 = \{wtw | w, t \in \{0, 1\}^+$ (15 points).

11. (20 points) Let $\Sigma = \{1, \#\}$ and consider the language $Y = \{w | w = x_1 \# x_2 \# \ldots \# x_k$ for $k \geq 0$, each $x_i \in 1^*$, and $x_i \neq x_j$ for $i \neq j\}$. Prove that $Y$ is not regular.

12. (20 points) For each of the following languages give the Minimum Pumping Length (MPL) and justify your answer:

    (a) (5 points) $L_1 = 001 \cup 0^* 1^*$.

(b) (5 points) $L_2 = 1^*01^*01^*$.

(c) (5 points) $L_3 = 1011$.

(d) (5 points) $L_4 = \Sigma^*$.

**Due date:** Wednesday 14 October 2009.