Extending the Lifetime of Wireless Networks While Ensuring Coverage

Sriram Pemmaraju Imran Pirwani

[sriram, pirwani]@cs.uiowa.edu

Department of Computer Science, The University of Iowa

Iowa City, USA

Energy Conservation

Conserving energy in sensor and wireless ad-hoc networks is critical.

> A single bulb on a strand of Christmas tree lights consumes about half a watt. Whether they use batteries, solar cells or gadgets that harvest energy from vibrations, as self-winding watches do, motes must operate on 1/10,000 of this power on an average.

[Culler, Mulder, *Scientific American*, 2004]

Techniques: topology control, sleep-activity scheduling,...

Sleep-Activity Scheduling

In high density regions, only a small fraction of the nodes (coordinators) need to be active for sensing or for communication. The rest can *sleep*.
[Chen et al., *ACM Wireless Networks Journal*, 2002.]

To maximize network lifetime, the role of coordinators needs to be rotated among nodes.

A *sleep-activity schedule* assigns periods of sleep and activity to nodes so as to minimize energy consumption.

Dominating Sets, Domatic Partitions

A k-*dominating set* of a graph $G = (V, E)$ is a subset $D \subseteq V$ of vertices such that each $v \in V$ is at most k hops from some node in D.

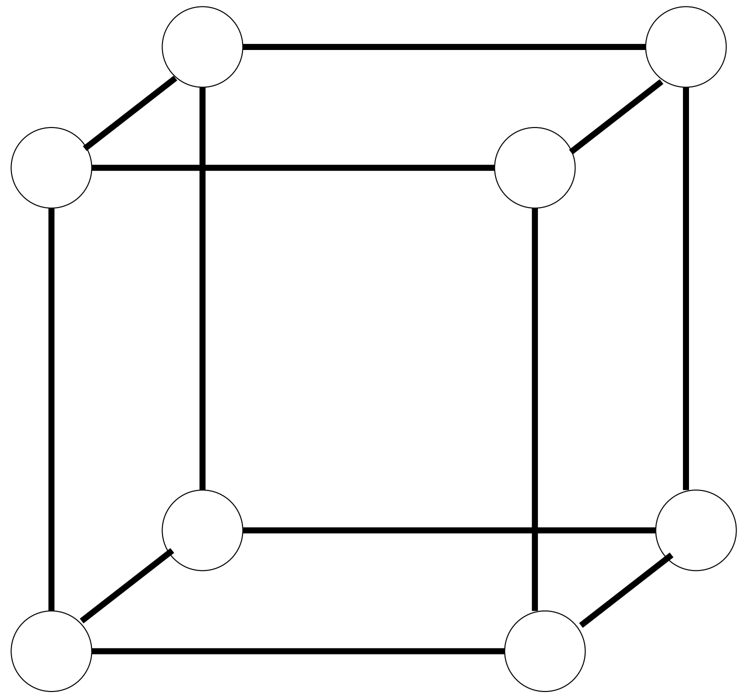
1-dominating set $\equiv$ dominating set.

A k-*domatic partition* is a partition $\mathcal{D} = \{D_1, D_2, \ldots, D_r\}$ of V such that each block D_i of $\mathcal{D}$ is a k-dominating set of G.

1-domatic partition $\equiv$ domatic partition.

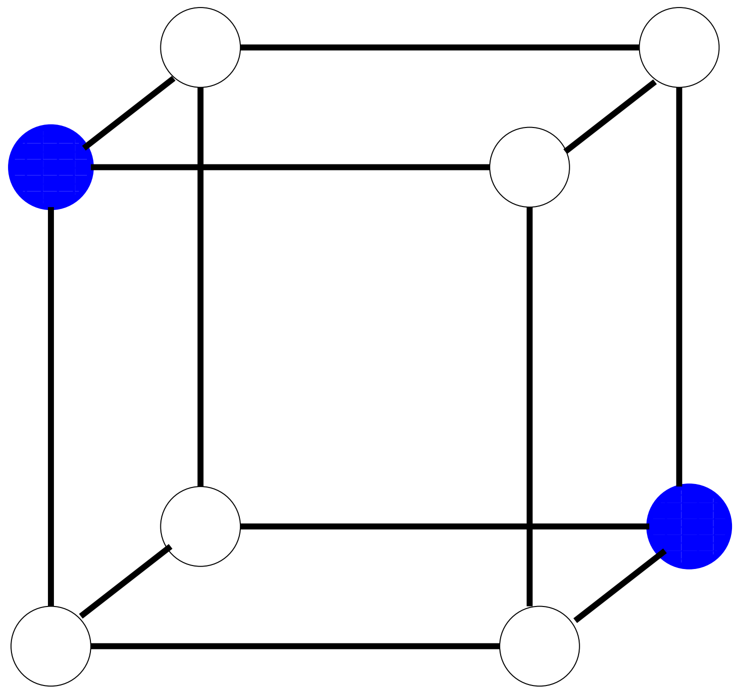
An Example

A domatic partition of the 3-dimensional hypercube



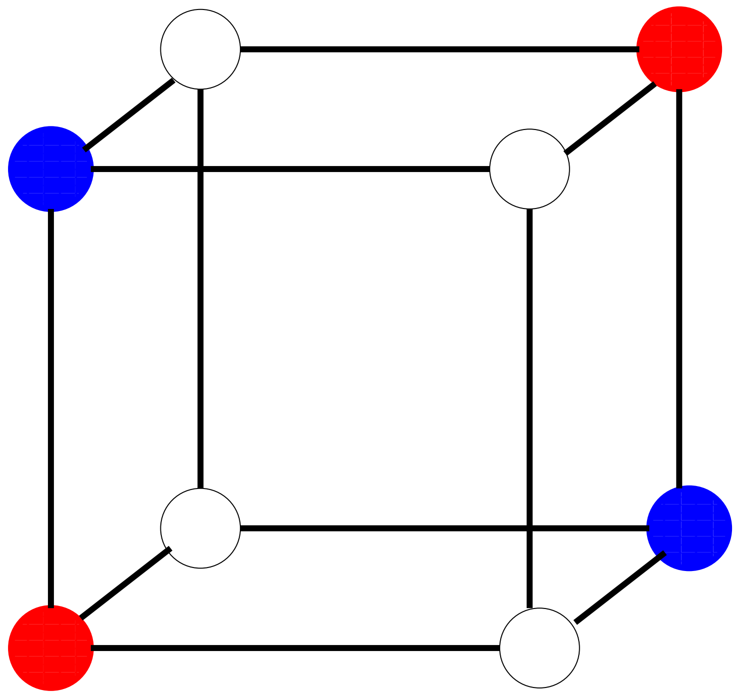
An Example

A domatic partition of the 3-dimensional hypercube



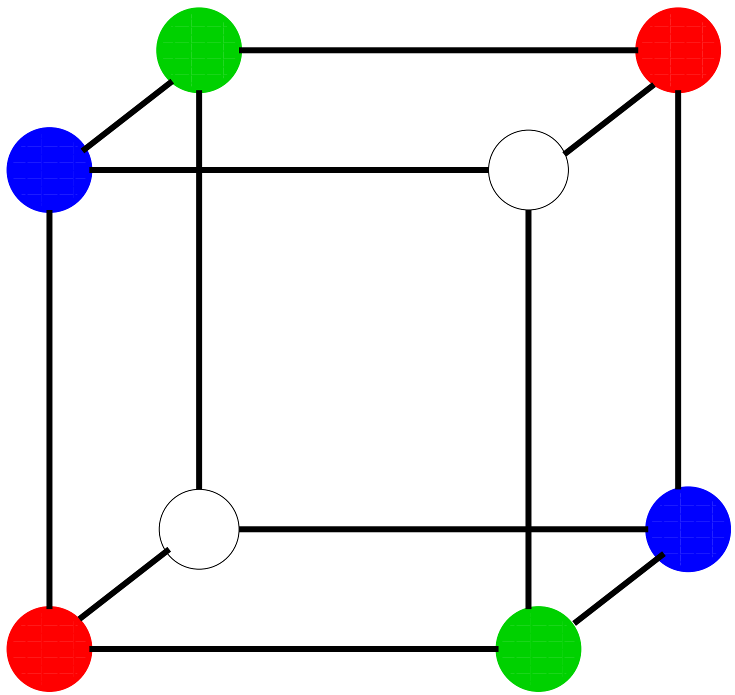
An Example

A domatic partition of the 3-dimensional hypercube



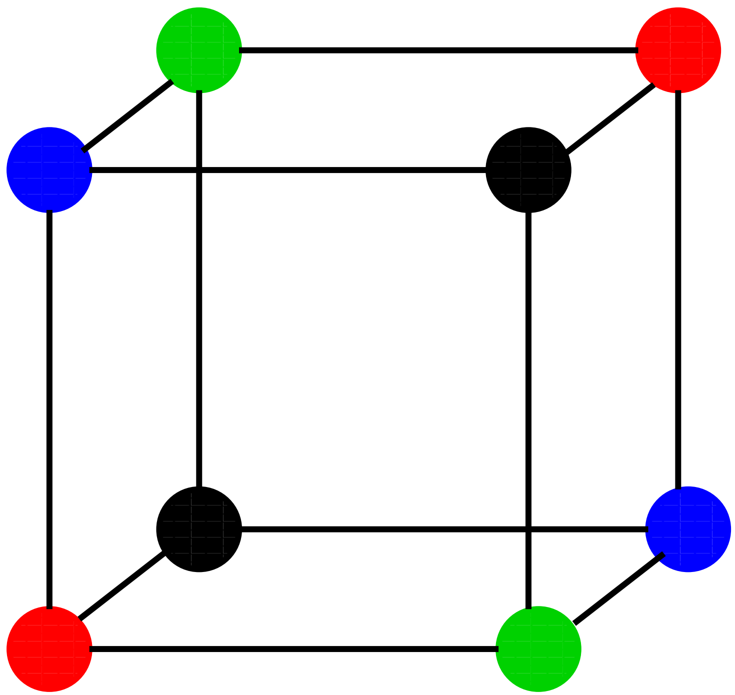
An Example

A domatic partition of the 3-dimensional hypercube



An Example

A domatic partition of the 3-dimensional hypercube



Hypercubes are *domatically full*.

The Connection

Given a domatic partition, $\mathcal{D} = \{D_1, D_2, \ldots, D_r\}$ of G:
Schedule:

- $[1, \ldots, T]$: nodes in D_1 are active,
- $[T+1, \ldots, 2T]$: nodes in D_2 are active,
- $\vdots$
- $[(r-1)\cdot T+1, \ldots, r\cdot T]$: nodes in D_r are active.

Over $r \cdot T$ time slots, each node is active for the fraction $1/r$ of time. Minimizing this fraction $\equiv$ maximizing size of k-domatic partitions.

Why k-domatic partitions?

$\delta =$ the minimum vertex degree in G. Size of largest domatic partition $\leq \delta + 1$.

$\delta^k =$ the minimum size of a k-neighborhood in G. Size of largest k-domatic partition $\leq \delta^k + 1$.

As k increases:

- size of largest k-domatic partition increases and
- the coordinators have to do more work.

This trade-off may imply that optimal value of $k > 1$.

Maximum k-Domatic Partition

INPUT: $G = (V, E)$
OUTPUT: A k-domatic partition of G of maximum size.

- NP-complete, even for $k = 1$.
- Has a $\frac{1}{O(\log \Delta)}$-approximation
  [Feige et al., "Approximating the Domatic Number," *SIAM J. Comput.*, 2002]
- Has a distributed implementation that runs in $O(1)$ rounds and provides a $\frac{1}{O(\log n)}$-approximation
  [Moscibroda, Wattenhofer, "Maximizing the Lifetime of Dominating Sets," *WMAN*, 2005]

Basic Question

For reasonable wireless network **models**, are there **efficient** distributed algorithms, that can compute **near optimal** k-domatic partitions?

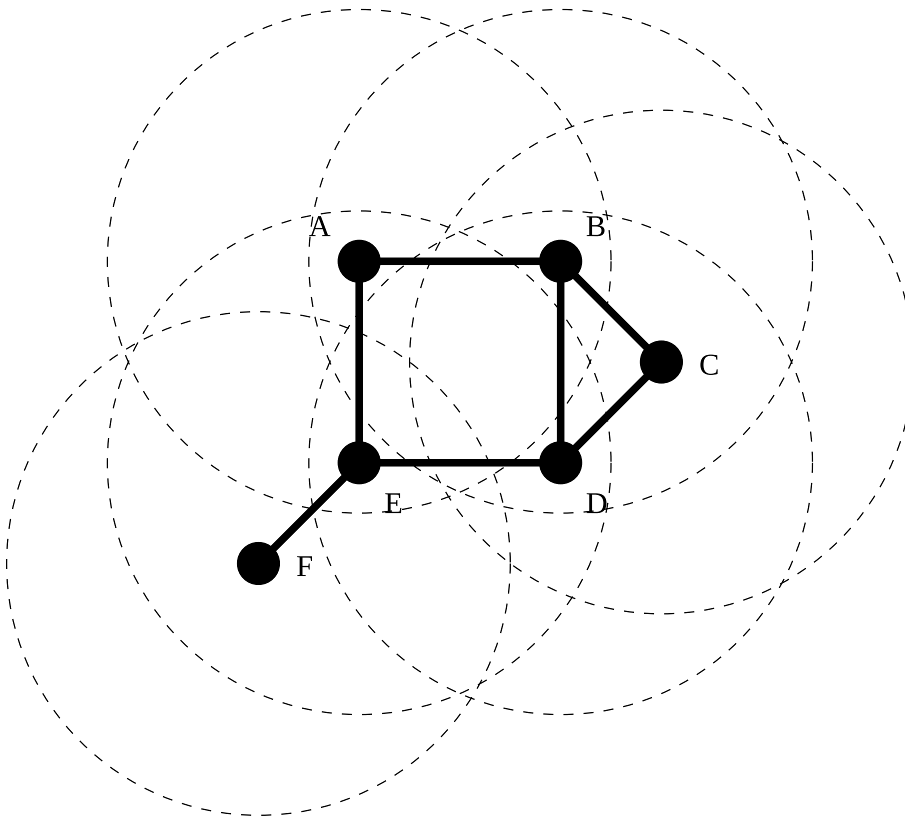
$$\begin{aligned}
\text{near optimal} &\equiv \frac{1}{O(1)}\text{-approximation} \\
\text{efficient} &\equiv O(polylog(n)) \text{ rounds} \\
\text{models} &\equiv \text{Euclidean unit ball graphs} \\
& \text{Doubling unit ball graphs} \\
& \text{Growth bounded graphs}
\end{aligned}$$

Euclidean Unit Ball Graphs

Nodes in G reside in d-dimensional Euclidean space $\mathbb{R}^d$

$$\{u, v\} \in E \Leftrightarrow |uv| \leq 1.$$

When $d = 2$, G is a *unit disk graph* (UDG).



Results

Model	Rounds
Euclidean UBGs (with location info.)	$O(1)$
Doubling UBGs (with distance info.)	$O(\log^* n)$
Growth bounded graphs (with only connectivity info)	$O(\log \Delta \cdot \log^* n)$

For any fixed $k \geq 2$, the constructed k-domatic partition has size at least $\frac{\delta_{k-1}}{c}$ for some constant c.

Implications

This is not a constant-approximation.

The size of the k-domatic partition is within a constant fraction of an optimal $(k-1)$-domatic partition.

We don't know how to construct a near optimal 1-domatic partition, but we can construct a 2-domatic partition whose size is within a constant fraction of an optimal 1-domatic partition.

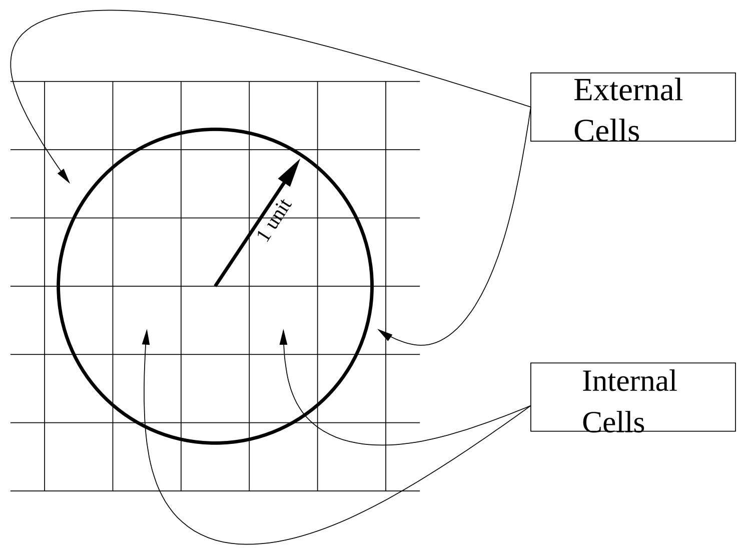
Algorithm

Input: A UDG $G = (V, E)$.
Output: A partition $\mathcal{D}$ of V such that every D_i is a k-dominating set.

1: Compute a partition of V into cliques $\{C_1, C_2, \ldots, C_t\}$ such that each k-neighborhood intersects a constant number of cliques.
2: **for all** clique $C_i : C_i \neq \emptyset$ **do**
3: Pick a vertex v; assign it to D_j; and remove it from C_i.
4: **end for**
{Repeat above loop for $j = 1, 2, \ldots$}
5: Output $\mathcal{D} = \{D_1, D_2, \ldots\}$.

Bounded Density Clique Partition

There are at most c, $\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$ cells that intersect a unit disk.



The k-neighborhood of a vertex v of a d-dimensional Euclidean UBG is contained in a d-dimensional radius-k ball. This intersects $O(k^d)$ hypercubes of fixed size. For fixed k and d, we get a clique partition of bounded density.

Clique Partition to Domatic Partition

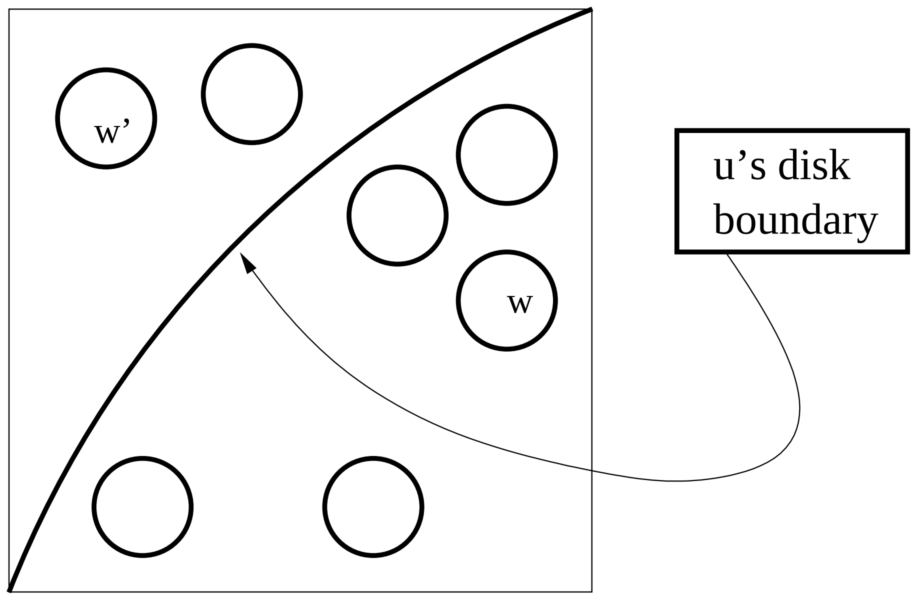
Let $D_1, D_2, \ldots$ be vertex subsets computed in Step (2) of algorithm.

CLAIM: For any r, $1 \leq r \leq (\delta_{k-1} + 1)/c_{k-1}$, the set D_r is a k-dominating set of G.

The proof for UDGs with $k = 2$, follows.

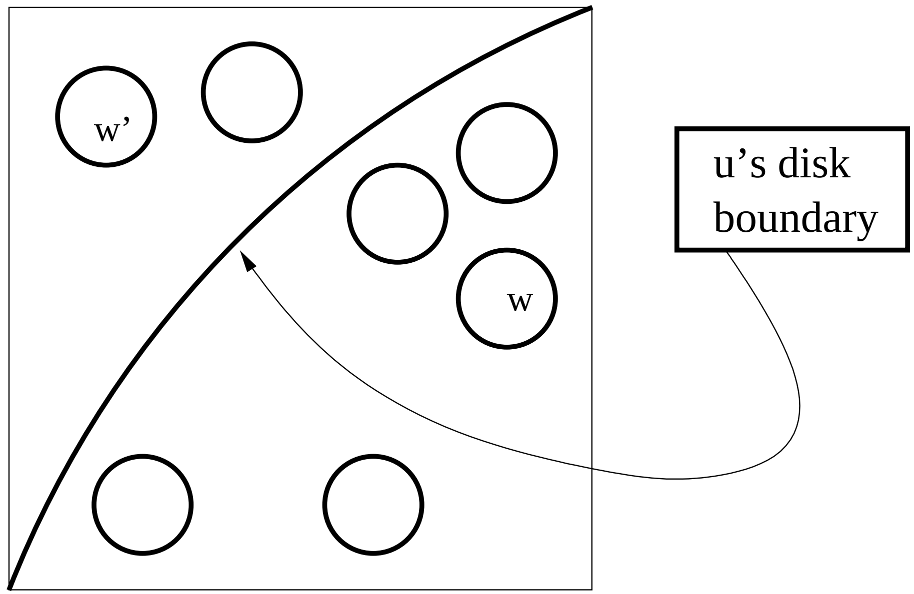
Proof

Say, D_r is not 2-dominating. Then there is a u that is more than 2 hops away from every vertex in D_r.



When D_r is constructed by the Algorithm (in Step (2)), all internal cells in u's disk are empty. Why?

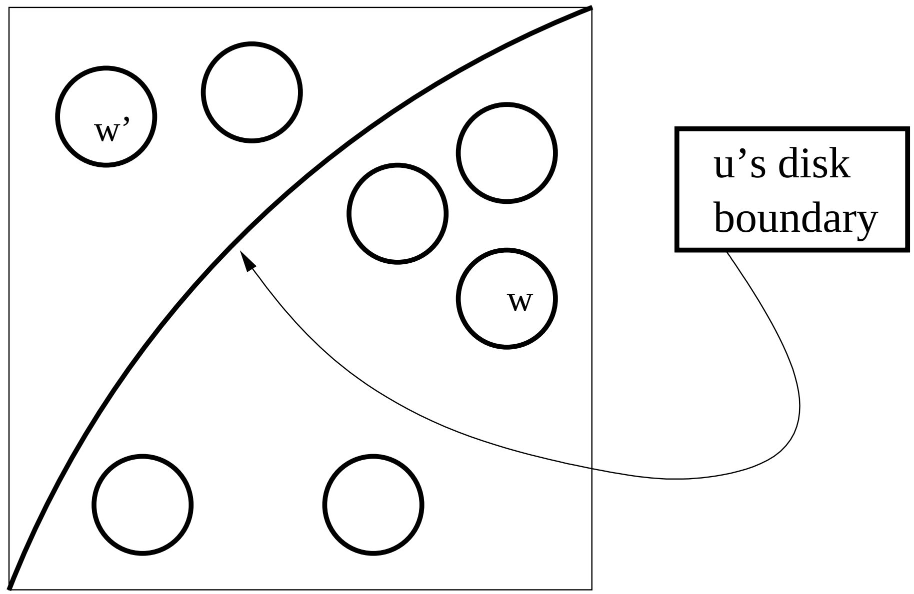
Proof (continued...)



Due to bounded density clique partition, each of $D_1, D_2, \ldots D_{r-1}$ removes a constant number (c for UDGs) of neighbors of u. Therefore, we get that,

$$|D_1 \cup D_2 \cup \cdots \cup D_{r-1}| \leq c \cdot (r-1).$$

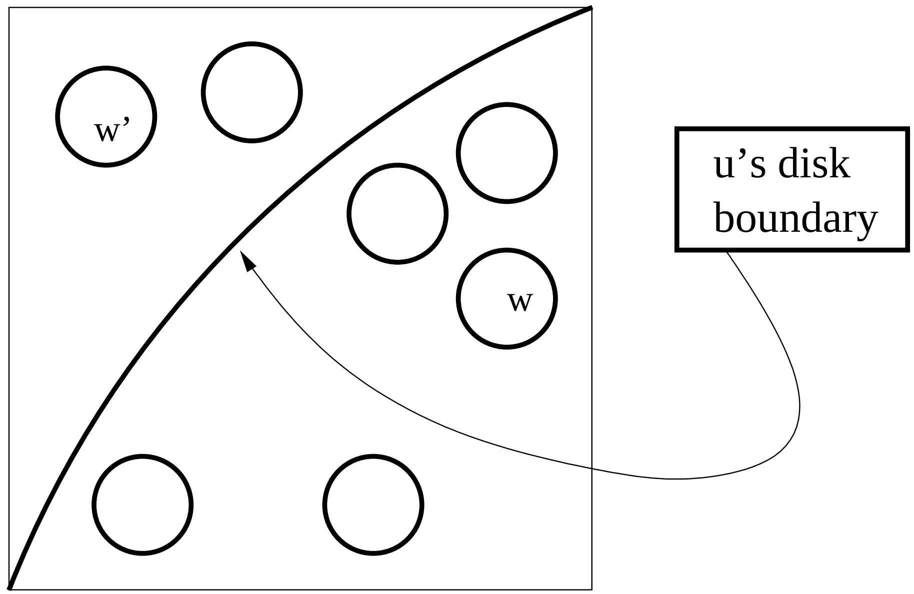
Proof (continued...)



But, $r \leq \frac{\delta+1}{c}$. This implies that,

$$c \cdot (r-1) < \delta + 1 \leq \deg(u) + 1$$

Proof (continued...)



Since $c \cdot (r-1) < \deg(u) + 1$, there is at least one vertex w in C_l that is in the disk of u that has not yet been colored. If w is placed in D_r then we are done. So, w' is placed in D_r. But, since C_l induces clique, w' 2-dominates u. A contradiction!

The rest of the story

- We can also construct a bounded density clique partition for UBGs in doubling metric spaces. This takes $O(\log^* n)$ rounds because it uses a maximal independent set (MIS) and MIS construction on UBGs in doubling metric spaces takes $O(\log^* n)$ rounds. [Kuhn et al., *PODC 2005*]

- For growth bounded graphs, our technique is slightly different. However, this also relies upon an MIS construction which takes $O(\log \Delta \cdot \log^* n)$ rounds. [Kuhn et al., *DISC 2005*]

Open Questions

- Is there a $\frac{1}{O(1)}$-approximation to the Domatic Partition Problem for:
  - Unit Disk Graphs?
  - Doubling Unit Ball Graphs?
  - Growth Bounded Graphs?
- Greedily picking out small dominating sets is known to perform poorly for general graphs. How well does it perform for the above classes of graphs?

Thank you!

Questions?