The Role of Aspiration Level in Risky Choice: A Comparison of Cumulative Prospect Theory and SP/A Theory

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Abstract

In recent years, descriptive models of risky choice have incorporated features that reflect the importance of particular outcome values in choice. Cumulative prospect theory (CPT) does this by inserting a reference point in the utility function. SP/A (security-potential/aspiration) theory uses aspiration level as a second criterion in the choice process. Experiment 1 compares the ability of the CPT and SP/A models to account for the same within-subjects data set and finds in favor of SP/A. Experiment 2 replicates the main finding of Experiment 1 in a between-subjects design. The final discussion brackets the SP/A result by showing the impact on fit of both decreasing and increasing the number of free parameters. We also suggest how the SP/A approach might be useful in modeling investment decision making in a descriptively more valid way and conclude with comments on the relation between descriptive and normative theories of risky choice.

Formal models of decision making under risk can be found in three disciplinary guises. Until quite recently, almost all economists believed that decision makers both should and do select risks that maximize expected utility. In contrast, investment professionals have seen investors as selecting portfolios that achieve an optimal balance between risk and return. Psychologists, too, have explored expected utility theory and portfolio theory as possible descriptive models, and they have also developed original information processing models focused on how people choose rather than what people choose.

For the most part, these three disciplines have been isolated from one another, although psychologists have been more eclectic in their outlooks than others. Recently, however, both psychologists and economists have been exploring a nonlinear modification of the expected utility model that we term the "decumulatively weighted utility" model. At present, this model has not affected practice in finance, but we believe the model can also be applied at the level of the individual investor.

In what follows, we first describe decumulatively weighted utility generically and then present two specific psychological instantiations of the model, cumulative prospect theory, (Tversky & Kahneman, 1992), and SP/A theory (Lopes, 1987; 1990; 1995). Second, we test the ability of these two theories to account for the same set of data. Third, we suggest how these results might apply in the investment context. Finally, we comment on fitting and testing complex, nonlinear models.

Decumulatively Weighted Utility

In a nutshell, the expected utility model asserts that when people choose between alternative probability distributions over outcomes (i.e., lotteries or gambles), they should (the economic model) and do (the psychological model) make their choices so as to maximize a probability weighted (i.e., linear) average of the outcome utilities. Although much evidence supports the idea that averaging rules can model people's choices and judgments reasonably well in a variety of tasks (Anderson, 1981), the linearity assumption of expected utility theory has not fared so well. Less than a decade after the publication of von Neumann and Morgenstern’s (1947) axiomatization of expected utility theory, Allais (1952/1979) demonstrated that linearity failed in qualitative tests comparing choices in which “certainty” was an option to choices in which it was not.

For almost three decades afterwards, economists ignored these failures of linearity whereas psychologists assumed that they represented subject failures rather than model failures. In the 1980s, however, some economists began to take the failures seriously and to seek out variants of expected utility that could better account for behavior. An obvious candidate at the time was prospect theory (Kahneman & Tversky, 1979), but this model violated stochastic dominance, “an assumption that many theorists [were] reluctant to give up” (Tversky & Kahneman, 1992, p. 299). Instead, these economists began to explore the normatively more acceptable idea of decumulatively weighted utility. The first economic applications were proposed independently by Quiggen (1982) Allais (1986), and Yaari (1987), followed quickly by many others (Chew, Karni & Safra, 1987; Luce, 1988; Schmeidler, 1989; Segal, 1989).

The best way to understand decumulative weighted utility is to start with the structure of the expected value model:

$$EV = \sum_{i=1}^{n} p_i \, v_i \,, \qquad \text{Eq. 1}$$

in which the v_i are the n possible outcomes listed in no particular order and the p_i are the outcomes’ associated probabilities. Expected utility theory simply substitutes utility, $u(v)$, for monetary outcomes:

$$EU = \sum_{i=1}^{n} p_i \, u(v_i) \,. \qquad \text{Eq. 2}$$

Weighted utility models (e.g., prospect theory) substitute decision weights, $w(p)$ for probabilities,

$$WU = \sum_{i=1}^{n} w(p_i) \, u(v_i) \,, \qquad \text{Eq. 3}$$

but this is the move that leads to violations of stochastic dominance.

Decumulative weighted utility models recast the issue of transforming raw probabilities to one of transforming decumulative probabilities:

$$DWU = \sum_{i=1}^{n} h\left(\sum_{j=i}^{n} p_j\right)\left(u\left(v_i\right) - u\left(v_{i-1}\right)\right) \quad \text{Eq. 4}$$

$$= \sum_{i=1}^{n} h(D_i)\left(u(v_i) - u(v_{i-1})\right).$$

In such models, the v_i are ordered from lowest (worst outcome) to highest (best outcome). $D_i = \sum_{j=i}^{n} p_j$ is the decumulative probability associated with outcome v_i; that is, D_i is the probability of obtaining an outcome at least as high as outcome v_i. Thus, D_1 (the decumulative probability of the worst outcome, v_1) is 1 (you get at least that for sure) and D_{n+1} (the decumulative probability of exceeding the best outcome, v_n) is zero.

The function, *h*, maps decumulative probabilities onto the range (0,1) and so preserves dominance. It can also provide an alternative to using curvature in the utility function to model risk attitudes. For example, in expected utility theory, if $u(v)$ is a concave function of *v*, decision makers will prefer sure things to actuarially equivalent lotteries, a pattern termed “risk aversion.” Decumulative weighting can predict the same pattern even while assuming that $u(v) = v$ (a variant of decumulatively weighted utility that we call decumulative weighted value) by letting $h(D)$ be a convex function of *D*. Although the predicted behavior is the same, we call it “security-mindedness” in order to distinguish between utility-based and probability-based mechanisms.

The other major risk attitudes also have analogues in the decumulative weighted value (or utility) model. These are given in Table 1. Both models can accommodate risk neutrality (expected value maximizing behavior) and both can accommodate the rejection of sure things in favor of actuarially equivalent lotteries (termed “risk seeking” in expected utility theory and “potential-mindedness” by us). Both models can also predict the “cautiously hopeful” (our term) pattern first described by Markowitz (1959) in which subjects buy both insurance and lottery tickets, thereby paying premiums sometimes to gamble and other times to avoid gambling.

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Insert Table 1 about here

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<u>Cumulative Prospect Theory</u>

In cumulative prospect theory (CPT), Tversky and Kahneman (1992) reformulated the original prospect theory in terms of (de)cumulative weighted utility. The utility function, *u(v)*, was unchanged from the original, being concave (risk averse) for gains and convex (risk seeking) for losses, with the loss function assumed to be steeper than the gain function ($\lambda > 1$):

$$u(v) = \begin{cases} v^{\alpha} \text{ if } v \geq 0 \\ -\lambda(-v)^{\beta} \text{ if } v < 0 \end{cases} \quad \text{Eq. 5}$$

The decumulative weighting function was also taken to differ for gains and losses. For gains, the hypothesized function has an inverse S-shape that reinforces risk aversion for most lottery types but tends toward risk seeking for long shots (lotteries that have small probabilities of large prizes):

$$w^{+}(D) = \frac{D^{\gamma}}{\left(D^{\gamma} + (1 - D)^{\gamma}\right)^{1/\gamma}}. \quad \text{Eq. 6}$$

For losses, the weighting function is cumulative rather than decumulative and S-shaped rather than inverse S-shaped. It reinforces risk seeking for most lotteries but tends toward risk aversion for long shots (lotteries that have small probabilities of very large losses):

$$w^{-}(P) = \frac{P^{\delta}}{\left(P^{\delta} + (1 - P)^{\delta}\right)^{1/\delta}}. \quad \text{Eq. 7}$$

Thus, the utility functions and (de)cumulative weighting functions of CPT are largely (but not perfectly) mirror-imaged from gains to losses, producing what Tversky and Kahneman (1992) term “a four-fold pattern” in the predicted pattern of lottery preferences.

There are two general features of CPT that should also be noted here. The first is that CPT is based on a psychophysical principle of diminishing sensitivity from a reference point. In the case of utility, the reference point is usually assumed to be zero (the status quo). In the case of (de)cumulative weights, there are reference points at 0 and at 1. In both cases, the rate of change in the perceived magnitude (of value or of likelihood) is assumed to be greatest near the reference point and to diminish as one moves away.

The second feature of CPT is that it, like all its predecessors in the weighted value family, is a one-criterion model. Although there is much room in the model for psychological variables to operate, in the end, all these factors are melded into a single assessment of lottery attractiveness.

SP/A Theory

SP/A theory (Lopes, 1987; 1990; 1995) is a dual criterion model in which the process of choosing between lotteries entails integrating two logically and psychologically separate criteria:

$$SP/A = f[SP, A], \qquad \text{Eq. 8}$$

where SP stands for a security-potential criterion and A for an aspiration criterion.

The SP (security-potential) criterion is modeled by a decumulatively weighted value rule (i.e., the model is identical to Equation 4 except that the utility function is assumed to be linear[1]):

$$SP = \sum_{i=1}^{n} h(D_i)\left(v_i - v_{i-1}\right). \qquad \text{Eq. 9}$$

The decumulative weighting function, $h(D)$, has the form:

$$h(D) = wD^{q_s+1} + \left(1 - w\right)\left[1 - \left(1 - D\right)^{q_p+1}\right] \qquad \text{Eq. 10}$$

for both gains and losses. The equation is derived from the idea that subjects assess lotteries from the bottom up (a security-minded analysis), or the top down (a potential-minded analysis), or both (a cautiously hopeful analysis)[2]. The parameters q_s and q_p represent the rates at which attention to outcomes diminishes as the evaluation process proceeds up or down. The parameter, w, determines the relative weight of the S and P analyses. If $w = 1$, the decision maker is strictly security-minded. If $w = 0$, the decision maker is strictly potential-minded. If $0<w<1$, the decision maker is cautiously hopeful, with the degrees of caution and of hope depending on the relative magnitudes of w and $1-w$. Although Equation 10 omits subscripts on parameters for notational simplicity, SP/A theory follows CPT in allowing q_s, q_p, and w to assume different values for gains and for losses, moderating the relative importance of security and potential in the overall SP assessment. Unlike CPT, however, the decumulative weighting function of SP/A theory does not switch between inverse S-shaped for gains and S-shaped for losses.

The A (aspiration) criterion operates on a principle of stochastic control (Dubins & Savage, 1976) in which subjects are assumed to assess the attractiveness of lotteries by the probability that a given lottery will yield an outcome at or above the aspiration level, α:

$$A = p(v \geq \alpha) \qquad \text{Eq. 11}$$

For present purposes, we treat the aspiration level as crisp, which is to say, a discrete value that either is or is not satisfied. In principle, however, the aspiration level may be fuzzy: some outcomes may satisfy the aspiration level completely, others to a partial degree, and still others not at all. To model this, Equation 11 would need to incorporate a particular probability, p_i, according to the degree that its associated outcome, v_i, satisfies the aspiration level (Oden & Lopes, 1997).

SP/A theory and CPT share some significant psychological features: they both model the process by which subjects integrate probabilities and values by a (de)cumulative weighting rule, and they both include a point on the value dimension that has special significance to subjects (the reference point for CPT and the aspiration level for SP/A). Indeed, the aspiration level may be considered to be a kind of reference point. However, the theories differ three ways in how these features function.

The first difference is in how the reference point (or aspiration level) exerts its impact. In CPT, the reference point is incorporated into the utility function and influences subjects by marking an inflection point about which outcomes are first organized into gains and losses, and then scaled nonlinearly in accord with a principle of diminishing sensitivity. In SP/A theory, the aspiration level participates in a direct assessment of lottery attractiveness reflecting a principle of stochastic control and separate from the decumulatively weighted SP assessment. Because SP and A embody different criteria, each may favor a different lottery. When this happens, SP/A theory predicts conflict, a prediction that does not follow from single-criterion models such as CPT.

The second difference is that CPT predicts a four-fold pattern across gain preferences and loss preferences. Although some small imperfections in the symmetry of the pattern might obtain due to small differences in the value and weighting functions for gains and losses, the overall pattern should be one of reflection between gains and losses. SP/A theory, in contrast, allows considerable asymmetry between gains and losses. In the most commonly observed case, subjects appear to avoid risks strongly for gains but to be more-or-less risk neutral for losses (Cohen, Jaffray, & Said, 1987; Hershey & Schoemaker, 1980; Schneider & Lopes, 1986; Weber & Bottom, 1989). Protocols suggest that this is because security-minded or cautiously hopeful subjects set modest aspiration levels for gains, allowing the SP and A criteria to reinforce one another. For losses, however, the same subjects set high aspiration levels, hoping to lose little or nothing, and thereby setting up a conflict between the A and the SP criteria (Lopes, 1995).

The third difference is that SP/A theory predicts nonmonotonicities in preference patterns that depend on whether or not the aspiration level is guaranteed to be met (by boosting all outcomes above the aspiration level) or guaranteed not to be met (by pushing all outcomes below the aspiration level) no matter which lottery of a pair is chosen. For example, consider a cautiously hopeful decision maker choosing between $50 for sure versus a 50/50 chance of $100, else nothing. Suppose also that the decision maker wants to win “at least something.” Although the SP assessment could favor the long shot mildly, the A assessment would favor the sure thing strongly, leading to a choice of the sure thing. If $50 were added to all outcomes, however, (e.g., $100 for sure versus a 50/50 chance at $150, else $50) the A assessment would "drop out" (since both options satisfy the aspiration level with certainty) leaving the SP assessment to carry the day. CPT, in contrast, is qualitatively unaffected by cases in which outcomes are all pushed upward or downward so long as no outcomes cross the reference point. The experiments that follow use this third difference to distinguish between the two theories and test their abilities to account for subjects' choices among a set of multioutcome lotteries.

Experiment 1

Method

Stimuli and task. Subjects chose between actuarially equivalent pairs of five-outcome lotteries comprising three positive (gain) sets and three negative (loss) sets. The standard positive lotteries are shown in Figure 1. The tally marks represent lottery tickets yielding the outcomes shown at the left. Each of these lotteries has 100 tickets and each has an expected value of approximately $100. The names indicated for the lotteries are for exposition only and were not used with subjects.

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Insert Figure 1 about here
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Scaled positive and shifted positive lotteries were created by transforming the outcomes in the standard positive lotteries linearly. (Examples are shown in Figure 2.) To create the shifted lotteries, standard positive outcomes were increased by $50 (bringing the expected value of the shifted positive lotteries to $150). To create the scaled positive lotteries, standard positive outcomes were multiplied by 1.145 (bringing the expected value of the scaled lotteries to $114.50). The multiplicative constant was chosen to equate the maximum outcomes ($398) in the scaled and shifted sets.

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Insert Figure 2 about here
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Standard negative, scaled negative and shifted negative lotteries were created by appending a minus sign to the outcomes in the respective positive sets.

Design and subjects. Lotteries within stimulus set were paired in all possible combinations (${}_6C_2$ = 15 pairs per set) and pairs were arrayed vertically on sheets of US letter paper. Two replications were created for each set differing in the order of the lotteries on the page.

Pairs from the positive sets were randomized together (within replication) with the constraint that no particular lottery appeared on adjacent pages. Pairs from the negative sets were randomized similarly. Each replication consisted of 45 pairs (3 sets x 15 pairs per set).

The subjects for this experiment were 80 undergraduate students at the University of Wisconsin who served for extra credit in introductory psychology courses.

Procedure. Subjects were run in groups of two to four. Each subject was given a notebook containing practice materials and the randomized stimulus pairs. At the beginning of the experiment, subjects were shown how to interpret the positive lotteries and were told that the amount of prize money for each of the lotteries in a pair was the same. Then they were told that we were interested in their preferences for distributions (i.e., how the prizes are distributed over tickets) and were given three positive pairs for practice. Subjects were asked to indicate whether they would prefer the top lottery or the bottom lottery if they were allowed to draw a ticket from either for free and keep the prize.

Next, subjects were shown exemplars of negative lotteries and were told that these represented losses. They were then asked to indicate for a set of three more practice pairs which of each pair they would prefer if they were forced to draw a ticket from one or the other and pay the loss out of their own pockets.

Stimulus notebooks were divided into five sections, the first containing the practice pairs and the remaining four containing the four sets of stimulus pairs (two positive replications and two negative replications). Positive and negative replications were alternated with half of the subjects beginning with a positive replication and the other half beginning with a negative replication. Each set was preceded by a colored sheet announcing that “The next set of lotteries are all win [or loss] lotteries”. Subjects went through the notebooks at their own pace, indicating preferences for the top or bottom lottery by circling “T” or “B” on a separate answer sheet. The task took about an hour for most subjects. All choices were hypothetical.

Results and Discussion

The data from Experiment 1 are shown in Figure 3 for gains (left panel) and for losses (right panel). Lotteries are listed along the abscissas in the order of subject preferences for the standard lotteries. The data have been pooled over subjects, replications, and stimulus pair. Each data point represents the proportion of times the average subject chose the lottery out of the total number of times that the lottery was available for choice. Each lottery was presented 10 times (5 pairs x 2 replications) so that the maximum number of choices per subject for a given lottery was 10 and the minimum was zero.

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Insert Figure 3 about here

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The data reveal four patterns that are of special significance. (1) For both gains and losses, there are obvious main effects for lotteries [$F(5,395) = 93.08$ and 25.00, respectively, $p < .0001$ for both] as well as interactions between lottery and condition [$F(10,790) = 48.99$ and 8.09, respectively, $p < .0001$ for both]; (2) For both gains and losses, the data for standard and scaled stimuli are virtually identical [$F(5,395) = 1.925$, $p = .09$ and $F(5,395) = 0.602$, $p = .69$, respectively]; (3) The slopes of the preference functions for the standard and scaled stimuli are steeper for gains than for losses; and (4) the preference functions for the shifted stimuli are non-monotonically related to the preference functions for standard and scaled stimuli, especially for gains, and the pattern of non-monotonicity reverses between gains and losses. Preference for lower risk lotteries decreases for gains and increases for losses whereas preference for higher risk lotteries increases for gains and decreases for losses. As will be seen, these differences between preference functions for shifted lotteries and preference functions for standard and scaled lotteries are critical to disentangling the roles of decumulative (or cumulative) weighting and aspiration level in risky choice.

In what follows, we use the Solver function of Microsoft Excel) to fit both CPT (Tversky & Kahneman, 1992) and SP/A theory (Lopes, 1990; Oden & Lopes, 1997) to the data. Solver is an iterative curve fitting procedure that adjusts free parameters to optimize the fit of a model to a data set according to whatever criterion the user specifies. We used root-mean-squared-deviation (RMSD) between predicted and obtained. In order to lessen the possibility of finding only a local minimum, good practice requires starting with one's best guesses of parameter values and then, once a minimum is found, checking the fit by systematically altering parameter values and rerunning the program to make sure that a better fit cannot be found. The values we report are the best that we could find.

For both CPT and SP/A, we fit the models to the aggregate choice proportions (given in Table 2) for the two-alternative choice task that subjects performed and then pooled the predictions across choice pairs to obtain means (as are shown for the obtained data in Figure 3). Although we had too few replications to fit single subject data, visual inspection of single subject means revealed that the patterns of primary interest were evident at the single subject level, being especially clear for strongly security-minded subjects and somewhat attenuated for subjects whose preferences tended toward cautious hopefulness or potential-mindedness.

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Insert Table 2 about here

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Fitting CPT. As noted previously, CPT is a one-criterion model in which both values and probabilities are transformed psychologically during the lottery evaluation process. The utility function (Equation 5) has three parameters: α defines the curvature of the function for gains (or values above the reference point); β defines the curvature of the function for losses (or values below the reference point); and λ defines the relative slope of the two functions, with the loss function specified to be steeper than the gain function ($\lambda > 1$).

Weights in CPT also are defined separately for gains and for losses, as shown in Equations 6 and 7. For gains, the function is decumulative with a parameter, γ, regulating both the curvature and crossover point of the inverse S-shaped weighting function. For losses, the function is cumulative with an analogous parameter, δ, regulating curvature and crossover points.

CPT was fit simultaneously to the data for the three scaling conditions (standard, scaled, and shifted) and for both outcome types (gains and losses), estimating a single set of six parameter values. The fitting process can best be understood by reference to Table 3. Matrix A shows the pair choice data for the scaled positive pairs. Each entry is the proportion of times that subjects preferred the column lottery to the row lottery. Lotteries are ordered across the columns in descending order of preference and down the rows in ascending order of preference. Complementary pairs of entries sum to 1.00, e.g., entries (1,1) and (6,6) in which riskless (RL) and long shot (LS) lotteries are opposed. The value .500 is entered in the minor diagonal where opposing lotteries are identical. These pairs were not included in the stimulus set for obvious reasons.

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Insert Table 3 about here

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Matrix B gives the best fitting predictions of CPT for scaled positive pairs obtained iteratively by minimizing the root mean square deviation between obtained and predicted choice proportions. The utility and weighting functions of CPT were fit using the five value and weight parameters described above to estimate CPT attractiveness[3] values for the six various lotteries. These values are shown as the column and row headings in Matrix B.

The second and final step was to use the CPT attractiveness values as input to a pair-choice process that we modeled using the logistic function of CPT difference scores shown below[4]:

$$p(CPT_{1>2}) = \frac{1}{1 + e^{-k(CPT_1 - CPT_2)}} \qquad \text{Eq. 12}$$

The function predicts the proportion of times that lottery 1 is preferred to lottery 2 based on the two individual attractiveness values. The function has a single parameter, *k*, that is inversely related to the variance of the distribution of difference scores, CPT_1 - CPT_2. Although it might seem reasonable to allow CPT to fit separate *k* parameters for gains and for losses, the second *k* would be redundant with λ and would, in the present case, allow λ to fall below 1 (see Footnote 5.)

Figure 4 shows the best-fitting predictions of CPT to the data for gains (left panel) and for losses (right panel). Parameter values are in Table 2. Although the RMSD of .0810 is respectable for fitting 90 data points with six parameters, a comparison of predictions and data (Figure 3) reveals a qualitative discrepancy for the shifted gain lotteries. Although CPT is able to capture the general flattening of this preference function relative to the standard and scaled functions, all but the prediction for the long shot are monotonically decreasing. In contrast, subjects' preferences for the shifted rectangular, bimodal, and long shot lotteries were all greater than for the shifted short shot. Moreover, the nonmonotonicity that is induced for the shifted long shot comes at the expense of incorrectly predicting nonmonotonicity for the standard and scaled long shots as well.

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Insert Figure 4 about here

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A second issue concerns the CPT parameter values (see Table 3). Beginning with the parameters for the utility functions, note that although the function for gains is sharply curved (α = .426), the function for losses is close to linear (β = .942). Second, looking at the probability weights, note that the function for gains shows considerable nonlinearity (γ = .685) whereas the function for losses is again essentially linear (δ = .980). Finally, looking at λ, the parameter that determines the relative slopes of the utility functions for gains and losses, note that it has reached its floor value of 1.00[5]. Although the values of these parameters are consistent with the observed fact that the loss data are essentially linear and relatively shallow in slope, there is nothing in CPT that would lead one to expect this large asymmetry between gains and losses. Indeed, CPT is well-known for its prediction of reflection in preferences between gains and losses (i.e., the four-fold pattern).

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Insert Table 3 about here

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In all, then, CPT does reasonably well in fitting the data if one considers only RMSD. When one looks at qualitative effects, however, CPT fails with the shifted gains. Moreover, in order to get a reasonable fit, CPT must make use of parameter values that are inconsistent with the underlying psychophysical principle (diminishing sensitivity from a reference point) on which both utility and weighting functions are theorized to depend.

<u>Fitting SP/A theory</u>. As explained previously, SP/A theory proposes that risky choice involves two criteria, one based on a comparison of decumulatively weighted averages of probabilities and outcomes (the SP criterion) and the other based on a comparison of probabilities of achieving an aspiration level (the A criterion). The SP assessment process (Equation 4) has three parameters: q defines the degree of attention to different outcomes in assessments of security (q_s)and potential (q_p), whereas w defines the relative importance of security and potential assessments overall. In principle, all three parameters might differ between gains and losses. The A assessment process (Equation 5) has a single parameter, α, the aspiration level, which can also differ for gains (α^+) and losses (α^-). Although α might need to be fit as a free parameter in some cases (e.g., with continuous outcome distributions or with manipulated aspiration levels), our stimuli and choice conditions justified fixing α^+ at 1 and α^- at zero[6].

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Insert Table 5 about here

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In fitting SP/A theory to the data, we modeled the SP criterion and the A criterion separately (but not sequentially) and integrated the results into a final choice. The procedure is schematized in Table 5. Matrix A gives the scaled positive data and Matrix B gives the best-fitting predictions based on just the SP criterion. The column and row headings are estimated SP attractiveness values. We let w differ between gains (w^+) and losses (w^-) but, in order to hold our parameters to six, set $q_s = q_p = q$ and used the same single value for both gains and losses. The entries in the cells are choice proportions, $p(SP_{1>2})$, obtained by applying the logistic function to difference scores just as we did in fitting CPT:

$$p(SP_{1>2}) = \frac{1}{1 + e^{-k(SP_1 - SP_2)}} . \qquad \text{Eq. 13}$$

The parameter, k, is inversely related to the variance of the distribution of difference scores, $SP_1 - SP_2$

Matrix C shows best-fitting predictions based on just the A criterion. The row and column headings show the probability that the row (or column) lottery will yield a value that meets the aspiration level (e.g., the riskless gain lottery, column 1, guarantees a nonzero payoff whereas the peaked gain lottery, column 2, has only a .96 probability of a nonzero payoff). The table entries are choice proportions, $p(A_{1>2})$, obtained by submitting the A values to a relative ratio process having a parameter, t, $0 \leq t$ that controls contrast. Equation 14 shows the process for gains:

$$p(A_{1>2}) = \frac{A_1^{t^+}}{A_1^{t^+} + A_2^{t^+}} . \qquad \text{Eq. 14}$$

The equation for losses looks a little different but has the same structure:

$$p(A_{1>2}) = 1 - \frac{(1 - A_1)^{t^-}}{(1 - A_1)^{t^-} + (1 - A_2)^{t^-}} \qquad \text{Eq. 15}$$

$$= \frac{\left(1 - A_2\right)^{t^-}}{\left(1 - A_1\right)^{t^-} + \left(1 - A_2\right)^{t^-}} .$$

The difference between gains and losses reflects the fact that gains engender an approach/approach process based on relative lottery goodness, *A*, whereas losses engender an avoidance/avoidance process based on relative lottery badness, *1-A*. In other words, the probability of choosing Lottery 1 over Lottery 2 for gains reflects the degree to which Lottery 1 is <u>better</u> than Lottery 2, whereas the probability of choosing Lottery 1 over Lottery 2 for losses reflects the degree to which Lottery 2 is <u>worse</u> than Lottery 1.

Matrix D combines the $p(SP_{1>2})$ values and the $p(A_{1>2})$ values according to:

$$p\left(SP/A_{1>2}\right) = \frac{\left[p(SP_{1>2})p(A_{1>2})\right]^{1/2}}{\left[p(SP_{1>2})p(A_{1>2})\right]^{1/2} + \left[\left(1 - p(SP_{1>2})\right)\left(1 - p(A_{1>2})\right)\right]^{1/2}} . \quad \text{Eq. 16}$$

This rule (which is useful for cases in which both the domain and the range of the function are 0 to 1) displays both averaging properties [$p(SP/A_{1>2})$ lies between $p(SP_{1>2})$ and $p(A_{1>2})$] and Bayesian properties (the impact of an input value depends on its extremity). The exponent, 1/2, sets the weight of the two input quantities to be equal. Although one might imagine that this could be a free parameter in the model, our experience suggests that it shares variance with the SP and A parameters, muddying the fitting process when it is included.

Figure 5 shows the best fitting predictions of SP/A theory to the data for gains (left panel) and for losses (right panel). The RMSD of predicted to obtained is .0681. (relative to .0810 for CPT). As can be seen by inspection of the figures, SP/A has also done a better job of capturing the qualitative features of the data. In particular, SP/A predicts the nonmonotonic increases in preference for the shifted rectangular, bimodal, and long shot gain lotteries.

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Insert Figure 5 about here
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The parameter values for the SP/A model are in Table 6. Although the values are generally reasonable, the values for the SP component are not far from an expected value fit (w = .50, $q_s = q_p = 1$). We believe this reflects the simplifying constraints that we imposed on the parameters of the SP criterion. We shall have more to say about the matter in the final discussion.

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Insert Table 6 about here
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Experiment 2

It is sometimes thought that the opportunities for comparison offered by repeated measures designs create choice patterns that might not occur if subjects made only a single choice. The purpose of Experiment 2 was to replicate the main finding of Experiment 1 (i.e., that shifted lotteries are evaluated differently than standard or scaled lotteries) using a between subjects design. We also wanted to collect subjects' reasons for their choices. Because between-subject experiments are costly in terms of the required number of subjects, we used only positive lotteries.

Method

The stimuli were the 45 pairs comprising the standard, scaled, and shifted positive sets. Each pair was printed separately on a single sheet of US letter paper. Sheets were randomized and distributed at the beginning of experimental sessions to subjects who were participating in other related experiments. In a given session, different subjects had different pairs. Consequently, it was necessary to describe how to interpret the lotteries in very general terms, never mentioning particular outcomes or numbers of tickets.

As in Experiment 1, subjects were asked to mark which of the two lotteries they would prefer if they were allowed to draw a ticket from either for free and keep the prize for themselves. They were also asked to write a sentence or two explaining the basis for their preference.

A total of 433 subjects from the University of Wisconsin–Madison and the University of Iowa participated in the experiment for extra course credit. Six of these subjects indicated by their written responses that they had not understood how to interpret the lotteries, leaving 427 usable responses ranging over the 45 choice pairs.

<u>Results and Discussion</u>

The data from Experiment 2 are shown in Figure 6. Clearly, the means are noisier than the means from Experiment 1, a result one might anticipate not only from the different subjects contributing to each data point, but also from the reduced amount of data going into each data point (400 choices per data point in the within-subject case versus about 24 choices per data point in the between subject case). Despite the noise, however, a chi square analysis shows that the key differences between the three scaling conditions were replicated. Specifically, (1) the patterns of preferences for standard stimuli and scaled stimuli do not differ significantly from one another, $\chi^2(1) = 1.65$, $p > .05$; whereas (2) the pattern of preferences for shifted stimuli differs significantly from the overall pattern of preference for standard and scaled stimuli, $\chi^2(1) = 25.13$, $p < .001$.

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Insert Figure 6 about here

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Table 7 illustrates the reasons that subjects gave for their choices, taking the long shot (LS) and short shot (SS) lotteries as examples. (These are the lotteries that are shown in Figure 2.) Protocols for the scaled condition are on the left and for the shifted condition are on the right. In each set, the first four subjects (plain text) chose with the majority whereas the last two subjects (italic) chose with the minority.

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Insert Table 7 about here

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In the scaled condition, the majority of subjects (5 of 8) preferred the short shot to the long shot. Protocols 1 through 4 show clearly that such subjects are concerned with achieving a nonzero outcome. In terms of SP/A theory, the A criterion seems to be outweighing the SP criterion. Neither of the remaining two subjects seems particularly concerned with avoiding zero (the A criterion). Consequently, the extra high outcomes in the long shot have more impact (the SP criterion).

In the shifted condition, the majority of subjects (7 of 10) preferred the long shot to the short shot. In particular, protocols 7, 8, and 10 convey the sense that the $50 guaranteed outcome is good enough (the A criterion is fully satisfied for both lotteries) allowing the subjects to choose the slightly riskier long shot (the SP criterion). In contrast, protocol 11 suggests that the subject has adjusted his or her aspiration level upward, so that $50 is now equivalent to “not winning” (the A criterion) whereas protocol 12 reveals a subject who focused on the relative magnitudes of high and low outcomes, but placed more weight on the low outcomes (the SP criterion).

In sum, then, Experiment 2 confirms that preference patterns differ for shifted lotteries and for standard or scaled lotteries even when the data are gathered in a between-subjects design. Moreover, the protocols show, as SP/A theory predicts, that this result reflects differences in the relative impacts of SP and A criteria under the shifted and scaled or standard conditions. In the shifted condition, all lotteries satisfy the A criterion for gains and none satisfy it for losses, allowing the SP criterion to manifest itself more strongly in either case. In the scaled and standard conditions, however, there are large differences in the degree to which the A criterion is satisfied, reducing the importance of the SP criterion overall.

Discussion

The model comparison in Experiment 1 showed that, on six parameters, SP/A does a better job than CPT of fitting the present set of choice data. Not only is the RMSD for SP/A 16% smaller, the model also captures (as CPT does not) the nonmonotonic relation between preferences for shifted lotteries and preferences for standard and scaled lotteries. Experiment 2 reinforced this finding by replicating the critical nonmonotonicity for gain lotteries in a between subjects design. It also provided protocols confirming that the nonmonotonicity may arise because adding $50 to standard positive lotteries eliminates aspiration level as a consideration for most subjects, thus enhancing the impact of the decumulatively weighted SP criterion. In what follows, we discuss the implications of this result for modeling investment risk. We also provide comments on fitting complex models along with two instructive comparisons to the six-parameter SP/A model. We end with a discussion of the relation between descriptive and normative theories.

Risk Taking and Aspiration Level

It has often been pointed out that when people are in economic difficulty, they tend to take risks that they would avoid under better circumstances. This tendency appears among sophisticated managers in troubled firms (Bowman, 1980; 1982) as well as among unsophisticated subsistence farmers (Kunreuther & Wright, 1979). Experimental studies using managers as subjects have also confirmed the tendency toward risk-taking for losses, at least when ruin is not at issue (Laughhunn, Payne & Crum, 1980; Payne, Laughhunn & Crum, 1981). Standard thinking in investment theory would not lead one to expect risk taking in threatening situations. Instead, hard-pressed decision makers should value low risk over high expected return and choose accordingly.

The S-shaped utility function of prospect theory seems to provide an explanation for this paradoxical risk-taking: people take risks when they face losses because their utility function for losses is “risk seeking” (i.e., convex). Though one can criticize the circularity of the “explanation,” it at least predicts preferences better than the more standard assumption of uniform “risk aversion” (i.e., diminishing marginal utility). But predicting preferences is only half the story, especially when predictions fail, as they often do in experimental studies of preferences for losses with students (Cohen, Jaffray & Said, 1987; Hershey & Schoemaker, 1980; Schneider & Lopes, 1986; Weber & Bottom, 1989) as well as with managers (MacCrimmon & Wehrung, 1986). The other half of the story can be found in protocol data. Studies by Mao (1970) and by Petty and Scott (cited in Payne, Laughhunn & Crum, 1980) suggest that managers tend to define investment risk as the _probability_ of not achieving a target rate of return (that is to say, an aspiration level).

No one can doubt that expected return (i.e., expected or mean value) is a central and well-understood concept for managers, but the concept of risk is less well understood. In portfolio theory, for example, risk is usually equated with outcome variance (Markowitz, 1959) but this is not entirely satisfactory descriptively since it treats wins and losses alike. Other approaches to defining risk try to bypass this objection by restricting the variance computation to losses (i.e., the semivariance) or by computing risk as a probability weighted average of deviations below a target level (Fishburn, 1977). Few, however, have explored the possibility of modeling risk as the raw probability of not achieving an aspiration level. One who has is Manski (1988) who developed the idea in what he called a _utility mass model_. Another approach that incorporates raw probabilities comes from Weber (1988) who augmented an expectation model with weighted probabilities of winning, losing, and breaking even.

SP/A theory incorporates both notions, each in a separate criterion. On the SP side, a security-minded weighting function (or a cautiously hopeful function displaying more caution than hope) pays more attention to the worst outcomes than to better outcomes. On the A side, the model operates on the probability of achieving the aspiration level. Although normative models usually focus on a single criterion, descriptive models must go where subjects lead. In the case at hand, the subjects seem to be saying that they understand and use the term “risk” in both distributional and aspirational senses. For example, in Table 6, two subjects refer explicitly to risk. In Protocol 8, one subject uses the concept in the A criterion sense: risk is the _chance_ of winning less. In Protocol 10, however, another subject does not count chances, but rather focuses on differences in prize amounts, an SP-focused analysis.

Practitioners work at the boundary between normative and descriptive. Clients expect guidance (the normative function) in how to achieve their personal goals (the descriptive function). For the client who is concerned about not meeting a target return, there seems little point in discussing variance. It would seem better for the professional to recognize in a client's spoken desires the relevance of those mathematical rules that seem most applicable and, then, to explain in a simple fashion, properties of the rules that may not be self-evident.

Although much has been claimed since von Neumann and Morgenstern (1947) about the dire consequences of violating linearity, recent examination of alternative rules based on decumulative weighted utility and aspiration criteria (e.g., Manski, 1988; March, 1996; Yaari, 1987) suggests that these alternatives are neither better nor worse than maximizing expected utility. They are, however, different and seem to come closer to doing what people want done.

Pushing the Model Tests

In Experiment 1, we fit the CPT and SP/A models on the same number of free parameters even though SP/A theory could reasonably use several more. In order to better illuminate the roles of the various psychological components of the theory, we now bracket the six-parameter SP/A fit by comparing it to a zero-parameter fit and a ten-parameter fit.

The top panel of Figure 7 shows what happens when the SP criterion of SP/A theory is neutralized by setting its parameters to yield the expected value for all lotteries (w = .50, $q_s = q_p$ = 1.00), thus allowing the A criterion to dominate. We set the aspiration level here as we did previously: α gains > 0; α losses = 0). The choice rule is also zero-parameter, assuming that subjects choose whichever lottery has the higher value on the A criterion and is indifferent if lotteries tie on aspiration level.

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Insert Figure 7 about here

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The A criterion alone produces a reasonable quantitative fit, with an RMSD of .1206. Although it may seem surprising that such a simple mechanism does so well in fitting complex data given the very complicated models that have been favored for risky choice recently (including both CPT and SP/A), the result maps well onto the classic finding of information processing studies using duplex bets (Payne & Braunstein, 1971; Slovic & Lichtenstein, 1968) that probability of winning dominates the choice process. Still, the best that aspiration can do by itself with the shifted data is to fit a flat line. Aspiration alone also predicts a mirror symmetry (reflection in preferences) between gains and losses whereas the actual loss preference functions are much flatter than the gain preference functions for all three lottery types.

The bottom panel of Figure 7 shows what happens with a full, ten-parameter fit of SP/A theory. The fit (RMSD = .0484) is obviously much better than the six-parameter fit. What interests us more, however, is that removing constraints on the SP parameters reveals theoretically meaningful values (see Table 8). Whereas previously the SP criterion came very close to an expected value criterion, the new parameter values suggest important process differences between gains and losses. For gains, it appears that the bottom-up (security) evaluation is more important than the top-down (potential) evaluation whereas, for losses, the top-down (potential) evaluation appears more important than the bottom-up (security) evaluation. Similarly, differences in the *w* parameter also suggest that the importance of security is greater for gains than for losses ($w^+ > w^-$). Thus, SP/A parameters confirm the CPT-based intuition that subjects do, indeed, evaluate high-risk options more favorably for losses than for gains.

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Insert Table 8 about here

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There is, however, an important difference between the mechanisms used by CPT and SP/A to account for these differences between gains and losses. CPT's zero reference point provides the rationale for qualitative inversions of its utility function (from concave for gains to convex for losses) and its decumulative weighting function (from inverse S-shaped for gains to S-shaped for losses). Thus, CPT specifies that the value processing mechanisms and probability weighting mechanisms used by subjects differ qualitatively for gains and losses. SP/A, on the other hand, allows the relative attention paid to worst outcomes and best outcomes to shift as a function of domain (gains versus losses) and also allows the relative importance of the SP and A components to differ between gains and losses. But domain-mediated reference effects in SP/A theory are potentially applicable to a broader range of domain differences.

For example, Edwards and von Winterfeldt (1986) propose that a person's risk attitude may be different in different "transaction streams." Choices involving amounts in what they call the "quick cash" and "play money" streams (the former being what people have available in their wallets and the latter being money reserved for enjoyment) should be less risk averse (i.e., less security-minded) than choices involving "capital assets" and "income and fixed expenditures" streams. Similarly, MacCrimmon and Wehrung (1986) found that executives are more risk averse in making decisions about their own personal investments than they are about business investments. Shifts of these sorts in one's willingness to accept risk need not involve gain/loss shifts. Instead, they may involve only differences in outcome scale (large versus small transaction stream) or differences in real-world expectations or consequences (personal decisions versus business decisions). The parameters of SP/A theory, while restricting the decumulative weighting function to an inverse S-shape for both gains and losses, nevertheless allow for modeling this broader class of reference effects through differences in the relative attention paid to bad versus good outcomes in the SP assessment and through the relative importance accorded to SP and A assessments in the final choice.

The Importance of Normative Theory for Description and Vice Versa

In most of economics, expected utility theory remains the workhorse of academic research--which is to say, of normative research--despite its poor fit to data from psychological experiments. Recently, however, a number of economists have turned their attention to testing expected utility theory in laboratory settings involving stylized economic games. Up to now, this new enterprise has been doggedly empirical and intently focused on theoretically appropriate task instantiation and on experimental rigor and control. Despite this attention to detail, however, the predictions of the theory have frequently not been borne out, leaving the experimentalists with the not inconsiderable task of persuading their colleagues that the model's failures are meaningful and should not be overlooked (for reviews, see the essays by Smith, 1982; 1989; 1991; and the various chapters in Kagel & Roth, 1995). There has not, however, been a commensurate effort from these researchers to develop better theory, although Roth (1995, p. 18) has pointed out that at least some of those who ran the earliest economic experiments expected that experimental data would contribute to the development of both better descriptive theories and better normative theories.

Most economists view their discipline as one that deals with ideally rational behavior and, thus, attach little significance to discrepancies between what the theory predicts and what people actually do. Psychologists, on the other hand, view their task as one of predicting behavior and describing its cognitive sources in psychologically meaningful terms, whether or not that behavior is rational. The utility and probability weighting functions of CPT rest on perceptual concepts. The SP and A components of SP/A theory rest on attentional and motivational concepts. Thus, both theories provide a psychological grounding that allows each to appeal directly to intuitions via easily understood and compellingly named components.

Intuitiveness is not enough, however. Mathematical analysis of the sort pursued here is necessary to specify the quantitative mechanisms from which theoretical predictions flow and to confirm that it is these specific mechanisms that provide the best account of behavior. History shows that it is easy to conflate phenomena with explanations, especially when the explanations appeal to intuition. Thus, the phenomenon of risk aversion became conflated with the idea of diminishing marginal utility (concavity) because the intuition was powerful and, indeed, is accurate, that constant marginal gains or losses in assets are more noticeable to poor people than to rich people. Conflation of phenomena with explanation is especially hazardous to theoretical advancement in that it suppresses interest in psychologically important alternative explanations, such as the aspiration and decumulatively weighted utility mechanisms on which this paper has focused.

The model comparisons we presented pitted competing psychological mechanisms against one another while constraining them to the same number of free parameters. For the data set at hand, SP/A theory provided the better fit, both quantitatively and qualitatively. However, a more important comparison may reside in the relative strengths and weaknesses of the three different parameterizations of SP/A theory. On the one hand, the zero-parameter model, relying solely on the aspiration level mechanism, did surprisingly well in providing a rough fit to the data. That a mechanism as simple as this was overlooked as an alternative to more complicated accounts is testimony to the unhealthy power that the “best existing theory” has to stifle research into alternatives. On the other hand, the 6 and 10 parameter versions of SP/A theory show the necessity of the SP component for modeling preferences among the shifted lotteries and for capturing the relative flattening of preferences for loss lotteries. Although the possible contributions of aspiration level should not have been overlooked, theorists and experimentalists since Bernoulli have not been foolish in pursuing weighted utility models. Aspiration alone is simply too simple.

SP/A theory is a descriptive theory, through and through. Its dual choice criteria--the security-potential criterion and the aspiration criterion--are both included because each seems necessary to adequately capture human choices under risk. It is worth noting, however, that even though these two criteria are inconsistent with expected utility maximization except in special cases, the rationality of each has been defended recently on normative grounds, [e.g., see, Manski’s (1988) utility mass model and Yaari’s (1987) decumulatively weighted value model). Although there is still a great divide between normative and descriptive theories of risky choice, perhaps we are seeing the first evidence that descriptive research is finally, as Roth (1995, p. 22) put it, “speaking to theorists.”

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Footnotes

Requests for reprints should be sent to Lola Lopes at the College of Business Administration, University of Iowa, Iowa City, IA 52242 (Lola-Lopes@uiowa.edu). Deirdre Huckbody ran the subjects and coded the data as part of an independent study during her senior year at the University of Wisconsin.

[1]Most theorists assume that *u(v)* is nonlinear without asking whether the monetary range under consideration is wide enough for nonlinearity to be manifest in the data. We believe that *u(v)* probably does have mild concavity that might be manifest in some cases (as, for example, when someone is considering the huge payouts in state lotteries). But for narrower ranges, we prefer to ignore concavity and let the decumulative weighting function carry the theoretical load.

[2]We do not provide the derivation of the SP/A decumulative weighting function at this time because it is not relevant to the present focus. Interested readers can contact us for details.

[3]We use the term “attractiveness” to refer to individual lottery values. Others have used “strength-of-preference” to mean the same thing, but we prefer to distinquish between individual lottery assessments (attractiveness) and choices (or preferences) between lotteries.

[4]Although CPT is intended to be a theory of risky choice, the process that maps two or more individual lottery assessments onto choice has not been specified. The logistic function that we apply here and below is commonly used as the cumulative probability distribution function in statistical decision theory models of the two-alternative choice process (Luce & Galanter, 1963) and is presumed to be neutral with respect to its impact on CPT’s and SP/A’s ability to fit the qualitative features of the data.

[5]A somewhat improved RMSD (.0770) resulted when λ was allowed to drop to 0.400 (i.e., for the gain function to be considerably steeper than the loss function). Although this value is consistent with the observed fact that the standard and scaled gain data

are steeper than the standard and scaled loss data, the value is not consistent with CPT's oft-repeated claim that "losses loom larger than gains."

[6]Our assumption concerning the values of the aspiration level is analogous to Kahneman and Tversky's assumption (1979; Tversky & Kahneman, 1992) that the reference point of the utility function is at zero. Although we, as they, might sometimes want to modify this simplifying assumption, in the present case the lottery outcomes are spaced widely enough that minimum (or maximum) outcomes are good approximations for what might be "at least a small gain" or "no more than a small loss" in choices between more continuous lotteries.

Figure Captions

Figure 1. Standard positive stimulus set. The tally marks represent lottery tickets yielding the outcomes shown at the left. Each lottery has 100 tickets and an expected value of approximately $100.

Figure 2. Examples of stimuli from the scaled positive stimulus set and the shifted positive stimulus set. Scaled stimuli are produced from standard stimuli by multiplying outcomes by 1.145. Shifted stimuli are produced from standard stimuli by adding $50 to each outcome.

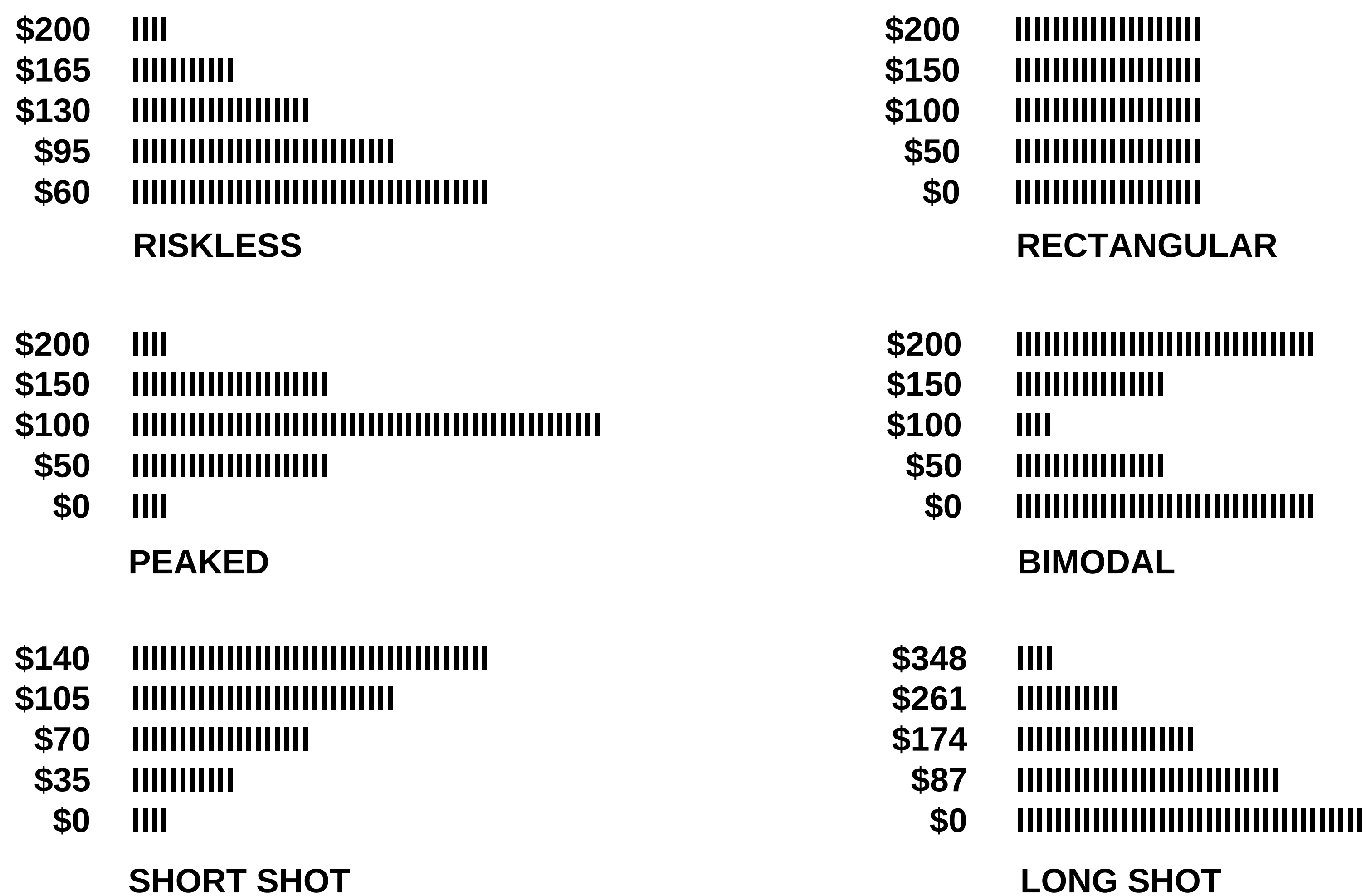
Figure 3. Data from Experiment 1 pooled over subjects, replications, and stimulus pair. Lotteries are listed along the abscissa in order of average subject preference for standard lotteries. Data are the proportion of occasions on which subjects chose a given lottery out of the 10 occasions on which that lottery was available for choice.

Figure 4. Predictions of CPT pooled over stimulus pair using the six parameter values shown in Table 4.

Figure 5. Predictions of SP/A theory pooled over stimulus pair using the six parameter values shown in Table 6.

Figure 6. Data from Experiment 2 pooled over subjects and stimulus pair. Data are the proportion of subjects choosing a given lottery out of the total number of subjects who had that lottery available for choice.

Figure 7. Top panel: SP/A predictions based on the A criterion alone with the SP criterion neutralized. Bottom panel: SP/A predictions based on the ten-parameter values shown in Table 8.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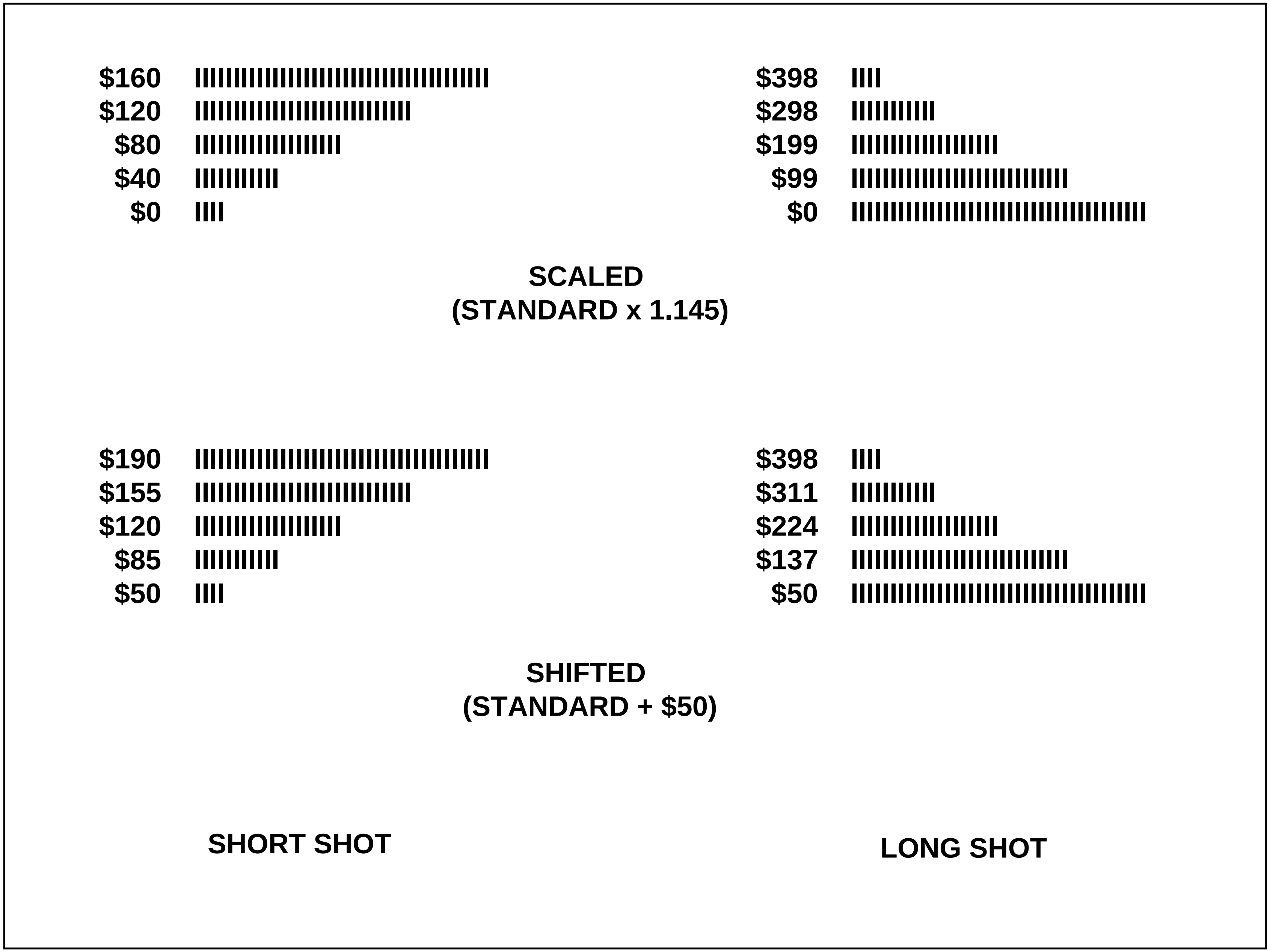
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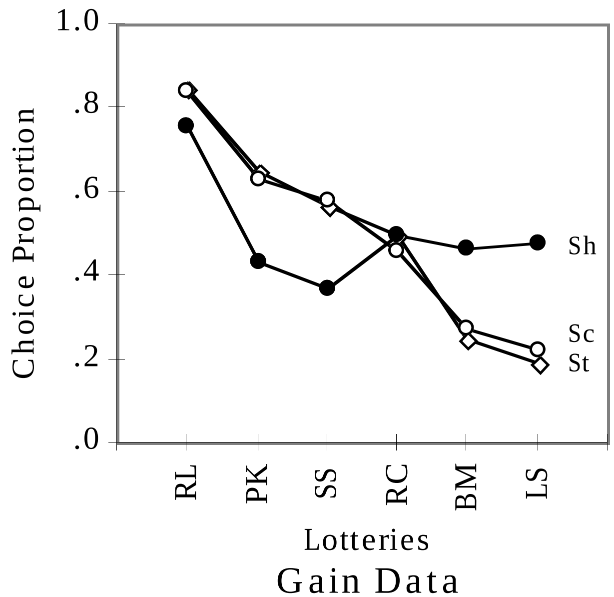

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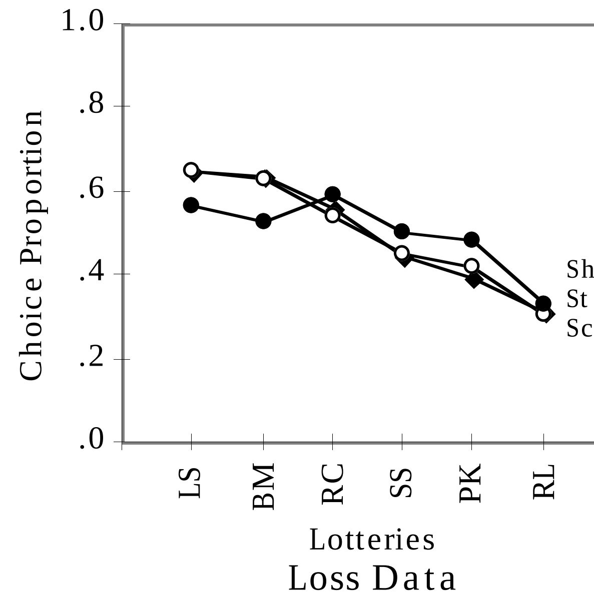

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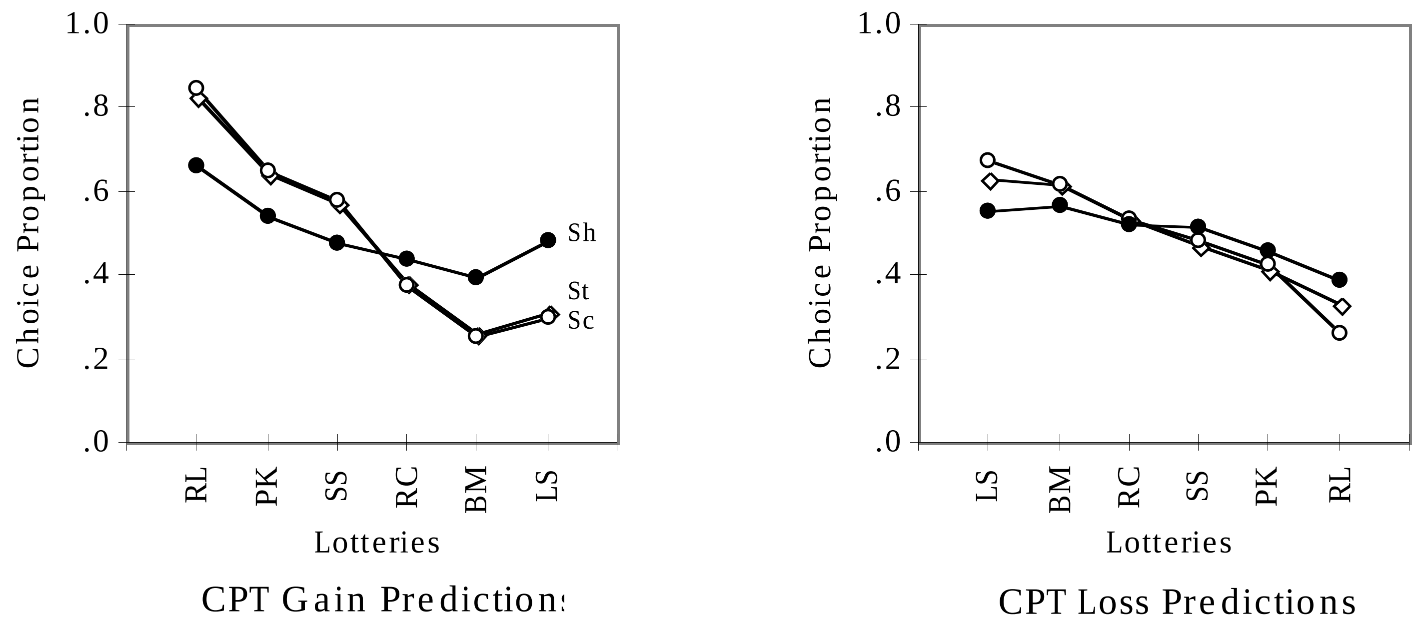
Lotteries
Loss Data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.6
.4
.2
.0
RL
PK
SS
RC
BM
LS
Sh
St
Sc
Lotteries
CPT Gain Predictions
Choice Proportion
1.0
.8
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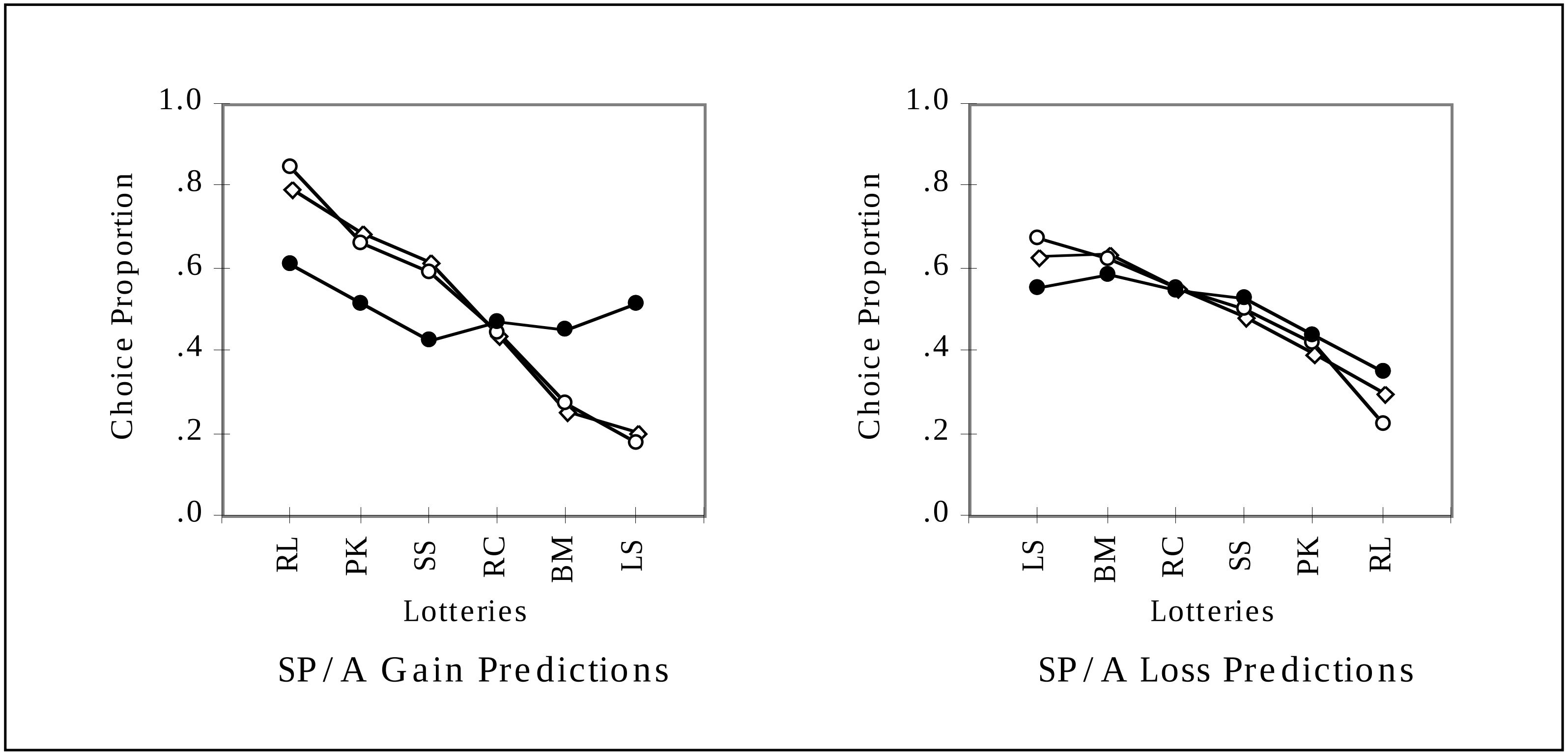
.0
LS
BM
RC
SS
PK
RL
Lotteries
CPT Loss Predictions


1.0
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Choice Proportion
RL
PK
SS
RC
BM
LS
Lotteries
SP/A Gain Predictions
1.0
.8
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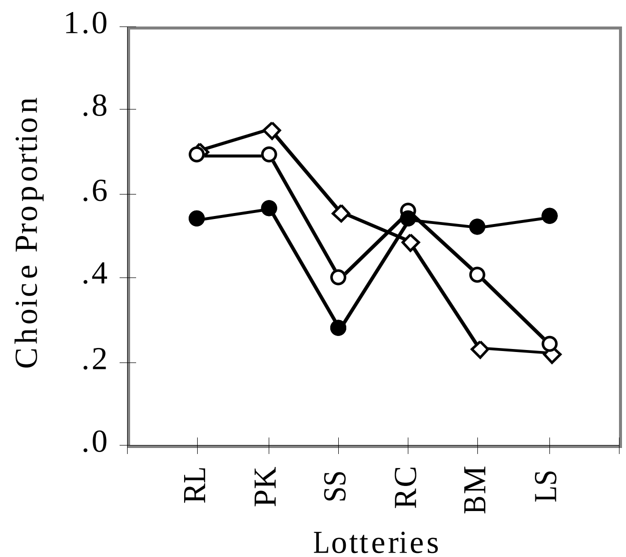
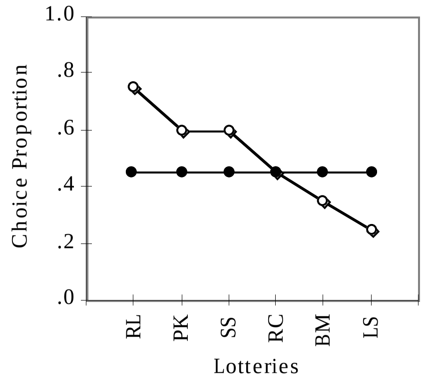
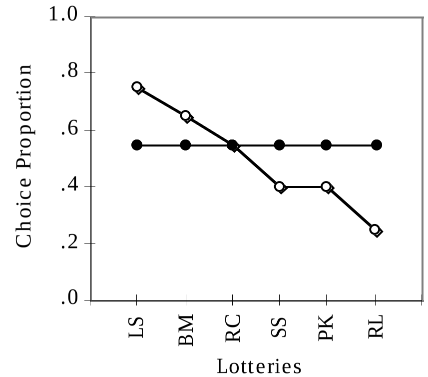
Choice Proportion
LS
BM
RC
SS
PK
RL
Lotteries
SP/A Loss Predictions



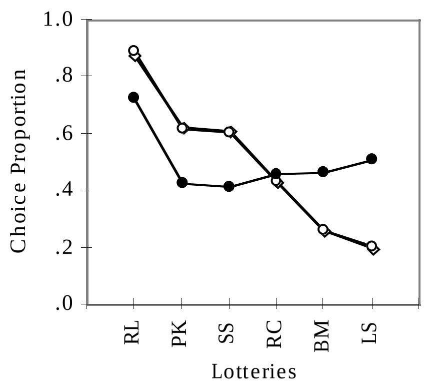
Between Subject Replication: Gains



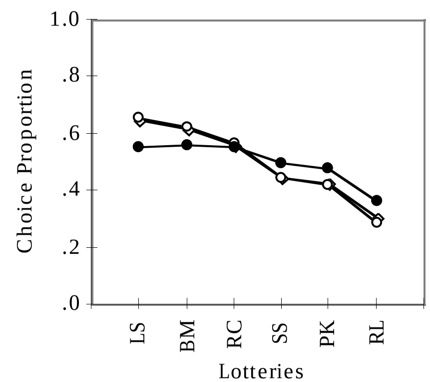
Gains: Aspiration Only



Losses: Aspiration Only



SP/A Gain Predictions (10 parameters



SP/A Loss Predictions (10 paramete

Table 1
Risk Attitudes in the
Expected Utility and Decumulative Weighted Value Models

Expected Utility Assumes h(D) = D		Decumulative Weighted Value Assumes u(v) = v	
Risk attitude	*u(v) is:*	*Risk attitude*	*h(D) is:*
Risk neutral	linear	Risk neutral	linear
Risk averse	concave	Security-minded	convex
Risk seeking	convex	Potential-minded	concave
Markowitz type	S-shaped	Cautiously hopeful	inverse S-shaped

Table 2

Choice Proportions

Standard lotteries

Gains	RL	PK	SS	RC	BM	LS	Losses	LS	BM	RC	SS	PK	RL
LS	.931	.844	.800	.869	.594	.500	RL	.781	.756	.731	.606	.556	.500
BM	.875	.819	.850	.806	.500	.406	PK	.713	.700	.656	.519	.500	.444
RC	.856	.719	.631	.500	.194	.131	SS	.575	.706	.613	.500	.481	.394
SS	.844	.588	.500	.369	.150	.200	RC	.606	.588	.500	.388	.344	.269
PK	.713	.500	.413	.281	.181	.156	BM	.569	.500	.413	.294	.300	.244
RL	.500	.288	.156	.144	.125	.069	LS	.500	.431	.394	.425	.288	.219

Scaled lotteries

Gains	RL	PK	SS	RC	BM	LS	Losses	LS	BM	RC	SS	PK	RL
LS	.856	.844	.813	.806	.563	.500	RL	.781	.763	.725	.644	.563	.500
BM	.856	.788	.813	.750	.500	.438	PK	.663	.675	.613	.500	.500	.438
RC	.863	.744	.650	.500	.250	.194	SS	.675	.631	.575	.500	.500	.356
SS	.844	.531	.500	.350	.188	.188	RC	.644	.563	.500	.425	.388	.275
PK	.769	.500	.469	.256	.213	.156	BM	.488	.500	.438	.369	.325	.238
RL	.500	.231	.156	.138	.144	.144	LS	.500	.513	.356	.325	.338	.219

Shifted lotteries

Gains	RL	RC	LS	BM	PK	SS	Losses	RC	LS	BM	SS	PK	RL
SS	.788	.625	.600	.625	.519	.500	RL	.725	.688	.650	.675	.613	.500
PK	.713	.550	.513	.588	.500	.481	PK	.606	.588	.525	.463	.500	.388
BM	.731	.575	.575	.500	.413	.375	SS	.544	.550	.538	.500	.538	.325
LS	.788	.519	.500	.425	.488	.400	BM	.594	.488	.500	.463	.475	.350
RC	.769	.500	.481	.425	.450	.375	LS	.488	.500	.513	.450	.413	.313
RL	.500	.231	.213	.269	.288	.213	RC	.500	.513	.406	.456	.394	.275

Note: Lotteries within each matrix are listed in descending order of preference across the columns and in ascending order of preference down the rows.

Table 3
Fitting CPT to the Data for Scaled Gain Pairs

Matrix A: Raw choice proportions

	RL	PK	SS	RC	BM	LS
LS	.856	.844	.813	.806	.563	.500
BM	.856	.788	.813	.750	.500	.438
RC	.863	.744	.650	.500	.250	.194
SS	.844	.531	.500	.350	.188	.188
PK	.769	.500	.469	.256	.213	.156
RL	.500	.231	.156	.138	.144	.144
Means	838	.628	.580	.460	.271	.224

Matrix B: Choice predictions based on CPT attractiveness values

	13.42	12.11	11.72	10.61	9.91	10.19
10.19	.916	.805	.756	.577	.449	.500
9.91	.931	.835	.792	.626	.500	.551
10.61	.889	.752	.694	.500	.374	.423
11.72	.779	.572	.500	.306	.208	.244
12.11	.725	.500	.428	.248	.165	.195
13.42	.500	.275	.221	.111	.069	.084
Means	.848	.648	.578	.374	.253	.299

Table 4
Parameter Values for CPT

Parameter	Value
α	0.551
β	0.970
λ	1.000
γ	0.699
δ	0.993
k	0.739

Table 5
Fitting SP/A Theory to the Data for Scaled Gain Pairs

Matrix A: Raw choice proportions

	RL	PK	SS	RC	BM	LS
LS	.856	.844	.813	.806	.563	.500
BM	.856	.788	.813	.750	.500	.438
RC	.863	.744	.650	.500	.250	.194
SS	.844	.531	.500	.350	.188	.188
PK	.769	.500	.469	.256	.213	.156
RL	.500	.231	.156	.138	.144	.144
Means	.838	.628	.580	.460	.271	.224

Matrix B: Choice predictions based on SP criterion

	114.86	114.00	113.66	113.96	113.97	113.88
113.88	.839	.537	.382	.510	.509	.500
113.91	.834	.528	.374	.501	.500	.491
113.91	.833	.527	.372	.500	.499	.490
113.60	.894	.652	.500	.628	.626	.618
113.98	.818	.500	.348	.473	.472	.463
114.86	.500	.182	.106	.167	.166	.161

Matrix C: Choice proportions based on A criterion

	1.000	0.960	0.960	0.800	0.680	0.620
0.620	.989	.984	.984	.917	.705	.500
0.680	.975	.963	.963	.823	.500	.295
0.800	.892	.848	.848	.500	.177	.083
0.960	.595	.500	.500	.152	.037	.016
0.960	.595	.500	.500	.152	.037	.016
1.000	.500	.405	.405	.108	.025	.011

Matrix D: Predictions combining SP and A criteria

	RL	PK	SS	RC	BM	LS
LS	.956	.895	.861	.773	.612	.500
BM	.933	.844	.797	.684	.500	.388
RC	.865	.714	.646	.500	.316	.227
SS	.779	.578	.500	.354	.203	.139
PK	.720	.500	.422	.286	.156	.105
RL	.500	.280	.221	.135	.067	.044
Means	.850	.662	.590	.446	.271	.181

Table 6
Parameter Values for SP/A Theory
(six parameter fit)

Parameter	Value
q	1.053
w^+	0.505
w^-	0.488
k	1.694
t^+	9.447
t^-	2.035

Table 7
Illustrative Protocols from Experiment 2

Scaled Lotteries (LS vs. SS)	Shifted Lotteries (LS vs. SS)
1. Have a greater chance of winning at least some money [in the SS]. The greatest chance is $160. [In the LS] lottery, your greatest chance is zero. (Picks SS.)	7. I picked [LS] because winning any amount of money would be exciting for me. If I picked $50 that would be great but if I picked any other number it automatically gives me more money in comparison with the other lottery choices.
2. There is a better chance to win money [in the SS] because there are more tally marks for the higher value of money than there is for $0. (Picks SS)	8. The amount of money to be won [in LS] is greater. Even though the chances of winning may be less, it's worth the risk. (Picks LS)
3. The odds of winning any amount of money here [in the SS] are a lot higher. Only 4 people out of 100 didn't get anything. (Picks SS)	9. The [SS] seems to be the better choice because the most tickets are for the larger prize amounts. But the [LS] has larger prizes. Your chances of winning more money seem to be greater here. (Picks LS)
4. There are more chances for winning money in the [SS]. The [LS] has a lot of tickets that have no monetary value. Even though the amounts are greater in the [LS], your chances are better in the [SS] to get a ticket worth money. (Picks SS.)	10. Excluding the extremes (the $190 top lottery prize in [SS] and the $50 lottery prize in [LS]) it appears that if you win, the prize will be for more money in [LS]. A better payoff for not that much more risk. (Picks LS)
5. I was not putting forth any money for this lottery, so even though the [LS] has a greater chance of getting $0—even if I was unfortunate—I lose nothing. But if I win, I win substantially more money. (Picks LS)	*11. I prefer the [SS] because there is more likelihood of winning. The best chance in the [LS] is $50 while the [SS] is $190. Although [with the LS] you can win more there is a greater chance of not winning.*
6. There is a better chance of winning more money [in the LS] and there is more money involved (higher prizes). (Picks LS.)	*12. There are less tickets for more money in the [LS]. I'd rather have a better chance for a little less money. My odds are better to get $190 [with SS] than $398 [with LS]. My odds are equal to get $190 [with SS] or $50 [with LS]. I opted for the $190 bracket. (Picks SS)*

Note: Protocols in italic are from subjects whose preferences went against majority preferences.

Table 8
Parameter Values for SP/A Theory
(ten-parameter fit)

Parameter	Value
q_s^+	372.07
q_p^+	64.37
q_s^-	4.86
q_p^-	16.58
w^+	0.837
w^-	0.003
k^+	.043
k^-	.023
t^+	10.000
t^-	2.070