

**22C : 031 Algorithms**  
**Sample Midterm**

This is closed book exam. You have an hour and fifteen minutes.

1. Arrange the following running times in a sequence in such a way that any element in the sequence is big-O of the succeeding element.

- (a)  $n \log n$
- (b)  $n^2$
- (c)  $n \log^3 n$
- (d)  $1.7^n$
- (e)  $n^{2.5}$

2. Show the sequence of the set of engaged pairs in an execution of the stable matching algorithm involving the three men  $m_1, m_2, m_3$  and the three women  $w_1, w_2, w_3$  with the following preferences:

- $m_1 : w_1 > w_2 > w_3$
- $m_2 : w_1 > w_3 > w_2$
- $m_3 : w_3 > w_1 > w_2$
- $w_1 : m_2 > m_3 > m_1$
- $w_2 : m_1 > m_2 > m_3$
- $w_3 : m_1 > m_3 > m_2$

3. Suppose that we call the recursive  $O(n^{\log_2 3})$  algorithm for multiplying two polynomials whose coefficients are represented by the following arrays:

- A: 1 7 2 4 6 3 5 2
- B: 3 0 7 6 3 7 6 9

What are the coefficient arrays that are passed to each of the three recursive calls made by the algorithm ?

4. Consider the  $O(n \log^2 n)$  algorithm we discussed in class (or the  $O(n \log n)$  algorithm in the textbook) for finding the closest pair in a given set of  $n$  points in the plane. Write down a recurrence for upper bounding the number of pairs of points whose distance is computed by the algorithm. What does this recurrence solve to? Derive as tight a bound as possible in the big-O notation.
5. The following algorithm takes as input an array  $A$  of  $n$  integers and a target integer  $t$ , and checks if there are two distinct elements in the array that add up to  $t$ .

```
flag := false
For i from 1 to n-1 do
  For j from i+1 to n do
    if (A[i] + A[j] == t) then flag := true
  endfor
endfor
Return flag
```

- (a) Give an asymptotically tight bound on the worst case running time of the algorithm as a function of  $n$ .
- (b) Describe a new algorithm for the same problem with an asymptotically faster running time. (This is a design question, you should consider attempting it after attempting the other questions.)