
99h:28019 28A80 42B10 46E30

Jorgensen, Palle E. T. (1-IA); **Pedersen, Steen** (1-WRTS)

Orthogonal harmonic analysis and scaling of fractal measures. (English. English, French summary)

C. R. Acad. Sci. Paris Sér. I Math. **326** (1998), *no.* 3, 301–306.

Let Ω be a measurable subset of $\mathbf{R}^d$ with finite Lebesgue measure m. A problem raised by I. E. Segal asks: For which sets Ω does the Hilbert space $L^2(\Omega, m)$ admit an orthonormal basis of the form $\{e_\lambda(x) = \exp(i2\pi\lambda\cdot x);\ \lambda\in\Lambda\}$ for some suitable subset Λ of $\mathbf{R}^d$?

In the present note, the authors continue to study related problems; namely, they consider pairs of measures (μ,ν) on $\mathbf{R}^d$ such that $F_\mu(\lambda) = \int e^{-i2\pi\lambda\cdot x} f(x)d\mu(x)$ induces an isometric isomorphism of $L^2(\mu)$ onto $L^2(\nu)$. It turns out that if μ is a finite measure, then ν is a counting measure with uniformly discrete support. The type of measure studied is of fractal nature and comes from special affine transformations of $\mathbf{R}^d$.

Paul Jolissaint (CH-NCH)