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Endomorphisms of $\mathcal{B}(\mathcal{H})$. (English. English summary)

Quantization, nonlinear partial differential equations, and operator algebra (*Cambridge, MA*, 1994), 93–138, *Proc. Sympos. Pure Math.*, 59, *Amer. Math. Soc.*, *Providence, RI*, 1996.

Let $\mathcal{B}(\mathcal{H})$ denote the algebra of all bounded linear operators on a separable, infinite-dimensional, complex Hilbert space H. If $\alpha: \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ is a unital endomorphism, the authors say that α is ergodic [resp., a shift] if $\{T \in \mathcal{B}(\mathcal{H}):\ \alpha(T) = T\}$ [resp., $\bigcap_{n=1}^{\infty} \alpha^n[\mathcal{B}(\mathcal{H})]$] consists only of scalar multiples of the identity operator. The (Powers) index of α is defined to be the extended integer n such that $\alpha[\mathcal{B}(\mathcal{H})]' \bigcap \mathcal{B}(\mathcal{H})$ is isomorphic to the factor of type I_n. Two unital endomorphisms α and β are conjugate if there exists an automorphism γ of $\mathcal{B}(\mathcal{H})$ such that $\alpha = \gamma\beta\gamma^{-1}$, and the authors say that α and β are approximately conjugate if α is in the norm-closure of the set of all endomorphisms that are conjugate to β.

This paper, semi-expository in nature, gives a deep and penetrating analysis of the conjugacy problem for endomorphisms of $\mathcal{B}(\mathcal{H})$. The work of M. Laca is highlighted: Laca showed that if $n \in \{2, 3, \cdots, \infty\}$, the set of conjugacy classes of all endomorphisms of index n can be parametrized by a natural Borel equivalence which is not countably separated (the same statement is valid for shifts and ergodic endomorphisms of index n). The authors give an independent proof of this theorem (for $n < \infty$) and describe an explicit set of labels for the conjugacy classes by exploiting the correspondence between endomorphisms of $\mathcal{B}(\mathcal{H})$ and representations of the Cuntz algebras which implement them. The Cuntz-algebra approach is also used to obtain interesting new results, among which are explicit examples of shifts with no invariant normal states, ergodic and clustering properties of endomorphisms of $\mathcal{B}(\mathcal{H})$, and a theorem which asserts that two such endomorphisms with the same index n, $2 \leq n < \infty$, are always approximately conjugate.

{For the entire collection see MR 96m:00020}.

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