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Coherent states of the q-canonical commutation relations. (English. English summary)

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The q-commutation relations $a(f)a^\dagger(g) - qa(g)^\dagger a(f) = \langle f, g\rangle \mathbf{1}$, $-1 \le q \le 1$, for f and g in a Hilbert space $\mathcal{H}$ provide a deformation from the canonical anti-commutation relations at $q = -1$ to the canonical commutation relations at $q = 1$, while at $q = 0$, $a(e_i)$ are co-isometries with orthogonal domains for e_i ranging over an orthonormal set in $\mathcal{H}$. These relations have been studied by several authors, and a q-deformed analogue of the Fock representation was constructed by D. I. Fivel [Phys. Rev. Lett. 65 (1990), no. 27, 3361–3364; MR 91m:81108], M. Bozejko and R. Speicher [Comm. Math. Phys. 137 (1991), no. 3, 519–531; MR 92m:46096] and D. B. Zagier [Comm. Math. Phys. 147 (1992), no. 1, 199–210; MR 93i:81105]. In the paper under review, for the renormalized relations $a(f)a^\dagger(g) - qa(g)^\dagger a(f) = (1-q)\langle f, g\rangle \mathbf{1}$, the authors construct coherent representations.

From the summary: "We consider representations generated from a vector Ω satisfying $a(f)\Omega = \langle f, \varphi\rangle\Omega$, where $\varphi \in \mathcal{H}$. We show that such a representation exists if and only if $||\varphi|| \le 1$. Moreover, for $||\varphi|| < 1$ these representations are unitarily equivalent to the Fock representation (obtained for $\varphi = 0$). On the other hand, representations obtained for different unit vectors φ are disjoint."

The universal C^*-algebra $\mathcal{E}_{\mathcal{H}}(q)$ generated by operators satisfying the q-canonical commutation relations was considered by the authors together with L. M. Schmitt [Pacific J. Math. 165 (1994), no. 1, 131–151; MR 95g:46116]. They showed that for $|q| < \sqrt{2} - 1 \approx 0.41$ and $d = \dim \mathcal{H} < \infty$, $\mathcal{E}_{\mathcal{H}}(q)$ is isomorphic to $\mathcal{E}_{\mathcal{H}}(0)$, the extension of the Cuntz algebra $\mathcal{O}_d$ by the compact operators. This in turn implies that the Fock representation is faithful on $\mathcal{E}_{\mathcal{H}}(q)$ for the same values of q and d. Now, whether the same holds for all $-1 < q < 1$ and all d is an interesting question. Moving in this direction, the authors show that $\mathcal{E}_{\mathcal{H}}(q)$ has a largest ideal, which is the kernel of the coherent representation π_φ when $||\varphi|| = 1$. The quotient of $\mathcal{E}_{\mathcal{H}}(q)$ by this ideal is defined to be the q-Cuntz algebra, $\mathcal{O}_{\mathcal{H}}(q)$ (isomorphic to $\mathcal{O}_d$ when $q = 0$). Thus a weaker version of the stability question is whether $\mathcal{O}_{\mathcal{H}}(q)$ is isomorphic to $\mathcal{O}_d$. Translating results of the reviewer and A. Nica [J. Reine Angew. Math. 440 (1993), 201–212; MR 94e:46117] into this context, the authors show that, for $-1 < q < 1$ and $d < \infty$, there is an embedding of $\mathcal{O}_d$ in $\mathcal{O}_{\mathcal{H}}(q)$, and this embedding is surjective, thus an isomorphism, for q in a slightly larger range $|q| <\approx$

0.44.

From the summary: “In the limiting cases $q = \pm 1$ we determine all irreducible representations of the relations, and characterize those which can be obtained via coherent states.” *Ken Dykema* (1-CA)