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The kernel of Fock representations of Wick algebras with braided operator of coefficients. (English. English summary)

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Let $\mathcal{W}(T)$ with $\mathcal{H}=\mathbb{C}^d$ be the Wick algebra with coefficients T_{ij}^{kl} (satisfying $T_{ji}^{lk}=\overline{T}_{ij}^{kl}$), i.e. the universal $*$-algebra generated by a_i, $1\le i\le d$, subject to the conditions $a_i^*a_j=\delta_{ij}1+\sum_{k,l=1}^d T_{ij}^{kl}a_la_k^*$, which can be realized as the quotient algebra

$$\mathcal{T}(\mathcal{H},\mathcal{H}^*)/\langle e_i^*\otimes e_j-\delta_{ij}1-\sum_{k,l=1}^d T_{ij}^{kl}e_l\otimes e_k^*\rangle$$

of the full tensor algebra $\mathcal{T}(\mathcal{H},\mathcal{H}^*)$ over $\mathcal{H}$ and its dual $\mathcal{H}^*$. Note that the canonical inclusions of $\mathcal{T}(\mathcal{H})$ and $\mathcal{T}(\mathcal{H}^*)$ in $\mathcal{T}(\mathcal{H},\mathcal{H}^*)$ when composed with the quotient map give rise to algebra embeddings of $\mathcal{T}(\mathcal{H})$ and $\mathcal{T}(\mathcal{H}^*)$ in the algebra $\mathcal{W}(T)$. The algebra representation λ_0, called the Fock representation, of $\mathcal{W}(T)$ on $\mathcal{T}(\mathcal{H})$ (the full tensor algebra over $\mathcal{H}$), uniquely determined by $\lambda_0(a_i)(e_{i_1}\otimes\cdots\otimes e_{i_n})=e_i\otimes e_{i_1}\otimes\cdots\otimes e_{i_n}$ and $\lambda_0(a_i^*)(1)=0$ via the defining commutation relations of $\mathcal{W}(T)$, is a $*$-representation with respect to a unique Hermitian sesquilinear form $\langle\cdot,\cdot\rangle_0$, called the Fock inner product, on $\mathcal{T}(\mathcal{H})$. A two-sided ideal $\mathcal{J}$ of $\mathcal{T}(\mathcal{H})\subset\mathcal{W}(T)$ is called a Wick ideal if $\mathcal{T}(\mathcal{H}^*)\otimes\mathcal{J}\subset\mathcal{J}\otimes\mathcal{T}(\mathcal{H}^*)$ in $\mathcal{W}(T)$. In this paper, it is proved that if $||T||\le 1$ for $T\colon\mathcal{H}\otimes\mathcal{H}\to\mathcal{H}\otimes\mathcal{H}$ satisfying the braid condition $T_1T_2T_1=T_2T_1T_2$ for $T_1=T\otimes\mathrm{id}_{\mathcal{H}}$ and $T_2=\mathrm{id}_{\mathcal{H}}\otimes T$ on $\mathcal{H}\otimes\mathcal{H}\otimes\mathcal{H}$, then the kernel of the Fock inner product $\langle\cdot,\cdot\rangle_0$ is the largest quadratic Wick ideal of $\mathcal{T}(\mathcal{H})$. It is also shown that for $-1<T\le 1$, the algebra $\mathcal{W}(T)$ has no nontrivial Wick ideals, which implies that the Fock representation λ_0 is faithful, and furthermore a known result is obtained as a corollary, namely, for $-1<T\le 1$, the Fock inner product $\langle\cdot,\cdot\rangle_0$ is strictly positive-definite.

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