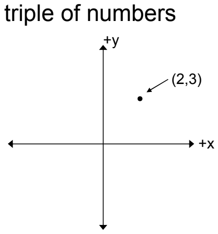
Relations

Chapter 10
22c:19
Hantao Zhang

Tuples

- In 2-dimensional space, it is a (*x*, *y*) pair of numbers to specify a location
- In 3-dimensional (1,2,3) is not the same as (3,2,1) – space, it is a (*x*, *y*, *z*) triple of numbers
- In *n*-dimensional space, it is a *n*-tuple of numbers
  - Two-dimensional space uses pairs, or 2-tuples
  - Three-dimensional space uses triples, or 3-tuples
- Note that these tuples are **ordered**, unlike sets
  - the *x* value has to come first



Cartesian products

- A Cartesian product is a set of all ordered 2-tuples where each "part" is from a given set
  - Denoted by A x B, and uses parenthesis (not curly brackets)
  - For example, 2-D Cartesian coordinates are the set of all ordered pairs **Z** x **Z**
    - Recall **Z** is the set of all integers
    - This is all the possible coordinates in 2-D space
  - Example: Given A = { a, b } and B = { 0, 1 }, what is their Cartiesian product?
    - C = A x B = { (a,0), (a,1), (b,0), (b,1) }

Cartesian products

- Note that Cartesian products have only 2 parts in these examples (later examples have more parts)
- Formal definition of a Cartesian product:

  A x B = { (a,b) | $a \in$ A and $b \in$ B }
- If |A| = m, |B| = n, how many pairs in A x B?
- Answer: mn.

4

Cartesian products

- All the possible grades in this class will be a Cartesian product of the set S of all the students in this class and the set G of all possible grades
  - Let S = { Alice, Bob, Chris } and G = { A, B, C }
  - D = { (Alice, A), (Alice, B), (Alice, C), (Bob, A), (Bob, B), (Bob, C), (Chris, A), (Chris, B), (Chris, C) }
  - The final grades will be a subset of this: { (Alice, C), (Bob, B), (Chris, A) }
    - Such a subset of a Cartesian product is called a relation (more on this later in the course)

5

Cartesian products

- There can be Cartesian products on more than two sets
- A 3-D coordinate is an element from the Cartesian product of **Z** x **Z** x **Z**

6

What is a relation

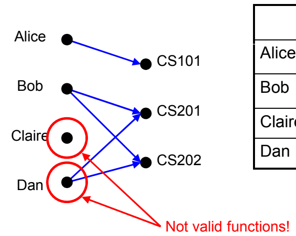
- Let *A* and *B* be sets. A binary relation *R* is a subset of $A \times B$
- Example
  - Let *A* be the students in a the CS major
    - *A* = {Alice, Bob, Claire, Dan}
  - Let *B* be the courses the department offers
    - *B* = {CS101, CS201, CS202}
  - We specify relation $R = A \times B$ as the set that lists all students $a \in A$ enrolled in class $b \in B$
  - R = { (Alice, CS101), (Bob, CS201), (Bob, CS202), (Dan, CS201), (Dan, CS202) }
- If |A| = m, |B| = n, how many different relations?
- Answer: 2^{mn}

More relation examples

- Another relation example:
  - Let *A* be the cities in the US
  - Let *B* be the states in the US
  - We define *R* to mean *x* is a city in state *y*
  - Thus, the following are in our relation:
    - (Philadelphia, PA)
    - (Portland, MA)
    - (Portland, OR)
    - etc.
- Most relations we know deal with ordered pairs of integers

Representing relations

We can represent relations graphically:

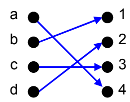


We can represent relations in a table:

	CS101	CS201	CS202
Alice	X		
Bob		X	X
Claire			
Dan		X	X

Relations vs. functions

- Not all relations are functions
- But consider the following function:



- All functions are relations!
- Any function f from A to B is relation Rf satisfying
  - For every x in A, there exists y in B, (x, y) in Rf.
  - If both (x, y), (x, z) are in Rf, then y = z.

10

When to use which?

- A function is used when you need to obtain a SINGLE result for any element in the domain
  - Example: sin, cos, tan
- A relation is when there are multiple mappings between the domain and the co-domain
  - Example: students enrolled in multiple courses

11

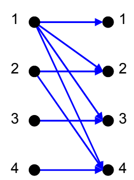
Relations on a set

- A relation on the set *A* is a relation from *A* to *A*
  - In other words, the domain and co-domain are the same set
  - We will generally be studying relations of this type

12

Relations on a set

- Let A be the set { 1, 2, 3, 4 }
- Which ordered pairs are in the relation R = { (a,b) | a divides b }
- R = { (1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4) }



R	1	2	3	4
1	X	X	X	X
2		X		X
3			X	
4				X

13

More examples

- Consider some relations on the set **Z**
- Are the following ordered pairs in the relation?

	(1,1)	(1,2)	(2,1)	(1,-1)	(2,2)
• R_1 = { (a,b) | $a \leq b$ }	X	X			X
• R_2 = { (a,b) | $a>b$ }			X	X	
• R_3 = { (a,b) | $a=\|b\|$ }	X			X	X
• R_4 = { (a,b) | $a=b$ }	X				X
• R_5 = { (a,b) | $a=b+1$ }			X		
• R_6 = { (a,b) | $a+b \leq 3$ }	X	X	X	X	

14

Relation properties

- Six properties of relations we will study:
  - Reflexive
  - Irreflexive
  - Symmetric
  - Asymmetric
  - Antisymmetric
  - Transitive

15

Reflexivity

- A relation is reflexive if every element is related to itself
  - Or, $(a,a) \in R$
- Examples of reflexive relations:
  - =, ≤, ≥
- Examples of relations that are not reflexive:
  - <, >

16

Irreflexivity

- A relation is irreflexive if every element is *not* related to itself
  - Or, $(a,a) \notin R$
  - Irreflexivity is the opposite of reflexivity
- Examples of irreflexive relations:
  - <, >
- Examples of relations that are not irreflexive:
  - =, ≤, ≥

17

Reflexivity vs. Irreflexivity

- A relation can be neither reflexive nor irreflexive
  - Some elements are related to themselves, others are not

18

Symmetry

- A relation is symmetric if, for every $(a,b) \in R$, then $(b,a) \in R$
- Examples of symmetric relations:
  - =, sibling(x, y), friend(x, y)
- Examples of relations that are not symmetric:
  - <, >, ≤, ≥

19

Asymmetry

- A relation is asymmetric if, for every $(a,b) \in R$, then $(b,a) \notin R$
  - Asymmetry is the opposite of symmetry
- Examples of asymmetric relations:
  - <, >, parent(x, y)
- Examples of relations that are not asymmetric:
  - =, friend(), ≤, ≥

20

Antisymmetry

- A relation is antisymmetric if $a=b$ whenever both $(a,b) \in R$ *and* $(b,a) \in R$.
  - Antisymmetry is *not* the opposite of symmetry
- Examples of antisymmetric relations:
  - =, ≤, ≥
- Examples of relations that are not antisymmetric:
  - friend()

21

Notes on symmetric relations

- A relation can be neither symmetric nor asymmetric
  - R = { (a,b) | a=|b| }
  - This is not symmetric
    - (4, -4) in R but (-4, 4) is not.
  - This is not asymmetric
    - (4, 4) is in R.
  - This is antisymmetric
    - If both (a, b) and (b, a) in R, then a = b.

22

Transitivity

- A relation is transitive if, for every $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$
- If $a < b$ and $b < c$, then $a < c$
  - Thus, < is transitive
- If $a = b$ and $b = c$, then $a = c$
  - Thus, = is transitive

23

Transitivity examples

- Consider Ancestor(x, y): x is an ancestor of y
  - Let Alice be Bob's parent, and Bob be Claire's parent
  - Thus, Alice is an ancestor of Bob, and Bob is an ancestor of Claire
  - Thus, Alice is an ancestor of Claire
  - Thus, Ancestor is a transitive relation
- Consider Parent(x, y)
  - Let Alice be Bob's parent, and Bob be Claire's parent
  - Thus, Alice is a parent of Bob, and Bob is a parent of Claire
  - However, Alice is *not* a parent of Claire
  - Thus, Parent() is *not* a transitive relation

24

Propertises of relations summary

	=	<	>	≤	≥
Reflexive	X			X	X
Irreflexive		X	X		
Symmetric	X				
Asymmetric		X	X		
Antisymmetric	X	X	X	X	X
Transitive	X	X	X	X	X

25

Matrix review

- We will only be dealing with zero-one matrices
  - Each element in the matrix is either a 0 or a 1

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

- These matrices will be used for Boolean operations
  - 1 is true, 0 is false

26

Matrix transposition

- Given a matrix **M**, the transposition of **M**, denoted $\mathbf{M}^t$, is the matrix obtained by switching the columns and rows of **M**

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\mathbf{M}^t = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$\mathbf{M}^t = \begin{bmatrix} 1 & 5 & 9 & 13 \\ 2 & 6 & 10 & 14 \\ 3 & 7 & 11 & 15 \\ 4 & 8 & 12 & 16 \end{bmatrix}$$

- In a "square" matrix, the main diagonal stays unchanged

27

Matrix join

- A *join* of two matrices performs a Boolean OR on each relative entry of the matrices
  - Matrices must be the same size
  - Denoted by the or symbol: $\vee$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\1&0&1&0\\1&0&1&0\end{bmatrix}\vee\begin{bmatrix}0&1&1&0\\0&1&1&0\\0&0&1&1\\1&1&0&0\end{bmatrix}=\begin{bmatrix}1&1&1&0\\0&1&1&0\\1&0&1&1\\1&1&1&0\end{bmatrix}$$

28

Matrix meet

- A *meet* of two matrices performs a Boolean AND on each relative entry of the matrices
  - Matrices must be the same size
  - Denoted by the or symbol: $\wedge$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\1&0&1&0\\1&0&1&0\end{bmatrix}\wedge\begin{bmatrix}0&1&1&0\\0&1&1&0\\0&0&1&1\\1&1&0&0\end{bmatrix}=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&1&0\\1&0&0&0\end{bmatrix}$$

29

Matrix Boolean product

- A *Boolean product* of two matrices is similar to matrix multiplication

$$c_{1,1}=a_{1,1}*b_{1,1}+a_{1,2}*b_{2,1}+a_{1,3}*b_{3,1}+a_{1,4}*b_{4,1}$$

  - Instead of the sum of the products, it's the conjunction (and) of the disjunctions (ors)

$$c_{1,1}=a_{1,1}\wedge b_{1,1}\vee a_{1,2}\wedge b_{2,1}\vee a_{1,3}\wedge b_{3,1}\vee a_{1,4}\wedge b_{4,1}$$

  - Denoted by the product symbol:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\1&0&1&0\\1&0&1&0\end{bmatrix}\begin{bmatrix}0&1&1&0\\0&1&1&0\\0&0&1&1\\1&1&0&0\end{bmatrix}=\begin{bmatrix}0&1&1&0\\0&1&1&0\\0&1&1&1\\0&1&1&1\end{bmatrix}$$

30

Relations using matrices

- List the elements of sets *A* and *B* in a particular order
  - Order doesn't matter, but we'll generally use ascending order
- Create a matrix

$$\mathbf{M}_R = [m_{ij}]$$

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

- We actually regard R as a boolean function over AxB: R(a, b) = true iff (a, b) in R.

31

Relations using matrices

- Consider the relation of who is enrolled in which class
  - Let A = { Alice, Bob, Claire, Dan }
  - Let B = { CS101, CS201, CS202 }
  - R = { (*a*,*b*) | person *a* is enrolled in course *b* }

R	CS101	CS201	CS202
Alice	X		
Bob		X	X
Claire			
Dan		X	X

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

32

Relations using matrices

- What is it good for?
  - It is how computers view relations
    - A 2-dimensional array
  - Very easy to view relationship properties
- We will generally consider relations on a single set
  - In other words, A and B are the same set
  - And the matrix is square

33

Reflexivity

- Consider a reflexive relation: ≤
  - One which every element is related to itself
  - Let A = { 1, 2, 3, 4, 5 }

$$\mathbf{M}_{\leq} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

If the center (main) diagonal is all 1's, a relation is reflexive

34

Irreflexivity

- Consider a reflexive relation: <
  - One which every element is *not* related to itself
  - Let A = { 1, 2, 3, 4, 5 }

$$\mathbf{M}_{<} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

If the center (main) diagonal is all 0's, a relation is irreflexive

35

Symmetry

- Consider an symmetric relation *R*
  - One which if *a* is related to *b* then *b* is related to *a* for all (*a*,*b*)
  - Let A = { 1, 2, 3, 4, 5 }

$$\mathbf{M}_{R} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

- If, for *every* value, it is the equal to the value in its transposed position, then the relation is symmetric

36

Asymmetry

- Consider an asymmetric relation: <
  - One which if *a* is related to *b* then *b* is *not* related to *a* for all (*a*,*b*)
  - Let A = { 1, 2, 3, 4, 5 }

$$M_< = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- If, for every value and the value in its transposed position, if they are not both 1, then the relation is asymmetric
- An asymmetric relation must also be irreflexive
- Thus, the main diagonal must be all 0's

37

Antisymmetry

- Consider an antisymmetric relation: ≤
  - One which if *a* is related to *b* then *b* is *not* related to *a* unless *a*=*b* for all (*a*,*b*)
  - Let A = { 1, 2, 3, 4, 5 }

$$M_\leq = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- If, for every value and the value in its transposed position, if they are not both 1, then the relation is antisymmetric
- The center diagonal can have both 1's and 0's

38

Transitivity

- Consider an transitive relation: ≤
  - One which if *a* is related to *b* and *b* is related to *c* then *a* is related to *c* for all (*a*,*b*), (*b*,*c*) and (*a*,*c*)
  - Let A = { 1, 2, 3, 4, 5 }

$$M_\leq = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- If, for every spot (*a*,*b*) and (*b*,*c*) that each have a 1, there is a 1 at (*a*,*c*), then the relation is transitive
- Matrices don't show this property easily

39

Combining relations

- There are two ways to combine relations R_1 and R_2
  - Via Boolean operators
  - Via relation "composition"

Combining relations via Boolean operators

- Consider two relations $R_{\geq}$ and $R_{\leq}$
- We can combine them as follows:
  - $R_{\geq} \cup R_{\leq}$ = { (x, y) | x ≥ y OR x ≤ y }
    - That's all the numbers
  - $R_{\geq} \cap R_{\leq}$ = { (x, y) | x ≥ y AND x ≤ y }
    - That's the equal relation.
  - $R_{\geq} \oplus R_{\leq}$ = { (x, y) | x ≥ y OR x ≤ y, but not both }
    - That's the unequal relation
  - $R_{\geq} - R_{\leq}$ = { (x, y) | x ≥ y but not x ≤ y }
    - That's the relation >.
  - $R_{\leq} - R_{\geq}$ = { (x, y) | x ≤ y but not x ≥ y }
    - That's the relation <.
- Note that it's possible the result is the empty set

Combining relations: via Boolean operators

- Let: $\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \mathbf{M}_S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
- Join:

$$\mathbf{M}_{R \cup S} = \mathbf{M}_R \vee \mathbf{M}_S = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

- Meet:

$$\mathbf{M}_{R \cap S} = \mathbf{M}_R \wedge \mathbf{M}_S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Combining relations via relational composition

- Let *R* be a relation from *A* to *B*, and *S* be a relation from *B* to *C*
  - Let $a \in A$, $b \in B$, and $c \in C$
  - Let $(a,b) \in R$, and $(b,c) \in S$
  - Then the composite of R and S consists of the ordered pairs (a,c)
    - We denote the relation by $S \circ R$
    - Note that *S* comes first when writing the composition!

43

Combining relations: via relation composition

- Let:

$$\mathbf{M}_R = \begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{matrix} d & e & f \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \quad \mathbf{M}_S = \begin{array}{c} \\ d \\ e \\ f \end{array}\begin{array}{c} \begin{matrix} g & h & i \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{M}_{S \circ R} = \mathbf{M}_R \mathbf{M}_S = \begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{matrix} g & h & i \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

44

Relational composition

- Let *M* be the relation "is mother of"
- Let *F* be the relation "is father of"
- What is $M \circ F$?
  - If $(a, b) \in F$, then *a* is the father of *b*
  - If $(b, c) \in M$, then *b* is the mother of *c*
  - $(a, c) \in M \circ F$ means "a is the father of the mother of c".
  - Thus, $M \circ F$ denotes the relation "maternal grandfather"
- What is $F \circ M$?
  - If $(a,b) \in M$, then *a* is the mother of *b*
  - If $(b,c) \in F$, then *b* is the father of *c*
  - $(a, c) \in F \circ M$ means "a is the mother of the father of c".
  - Thus, $F \circ M$ denotes the relation "paternal grandmother"
- What is $M \circ M$?
  - If $(a,b) \in M$, then *a* is the mother of *b*
  - If $(b,c) \in M$, then *b* is the mother of *c*
  - Thus, $M \circ M$ denotes the relation "maternal grandmother"
- Note that *M* and *F* are not transitive relations!!!

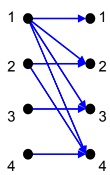
45

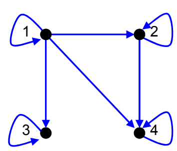
Relational composition

- Given relation R
  - $R \circ R$ can be denoted by R^2
  - $R^2 \circ R = (R \circ R) \circ R = R^3$
  - Example: M^3 is your mother's mother's mother.

Representing relations using directed graphs

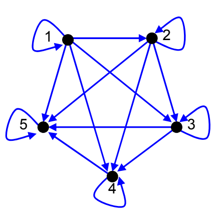
- A directed graph consists of:
  - A set V of vertices (or nodes)
  - A set E of edges (or arcs)
  - If (a, b) is in the relation, then there is an arrow from a to b
- Will generally use relations on a single set
- Consider our relation $R = \{ (a,b) \mid a \text{ divides } b \}$
- Old way:





Reflexivity

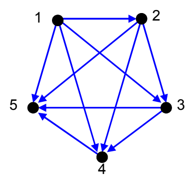
- Consider a reflexive relation: ≤
  - Every element is related to itself
  - Let A = { 1, 2, 3, 4, 5 }



If every node has a loop, a relation is reflexive

Irreflexivity

- Consider a reflexive relation: <
  - Every element is *not* related to itself
  - Let A = { 1, 2, 3, 4, 5 }

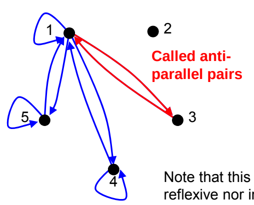


If every node does *not* have a loop, a relation is irreflexive

49

Symmetry

- Consider an symmetric relation *R*
  - If *a* is related to *b* then *b* is related to *a* for all (*a*,*b*)
  - Let A = { 1, 2, 3, 4, 5 }



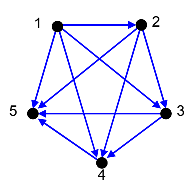
- If, for every edge, there is an edge in the other direction, then the relation is symmetric
- Loops are allowed, and do not need edges in the "other" direction

Note that this relation is neither reflexive nor irreflexive!

50

Asymmetry

- Consider an asymmetric relation: <
  - If *a* is related to *b* then *b* is *not* related to *a* for all (*a*,*b*)
  - Let A = { 1, 2, 3, 4, 5 }



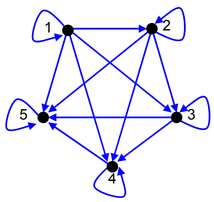
- A digraph is asymmetric if:

1. If, for every edge, there is *not* an edge in the other direction, then the relation is asymmetric
2. Loops are *not* allowed in an asymmetric digraph (recall it must be irreflexive)

51

Antisymmetry

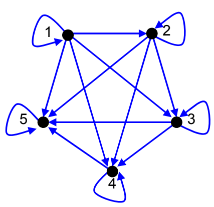
- Consider an antisymmetric relation: ≤
  - If *a* is related to *b* then *b* is *not* related to *a* unless *a*=*b* for all (*a*,*b*)
  - Let A = { 1, 2, 3, 4, 5 }



- If, for every edge, there is *not* an edge in the other direction, then the relation is antisymmetric
- Loops are allowed in the digraph

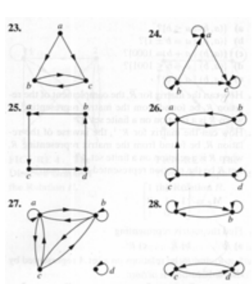
Transitivity

- Consider an transitive relation: ≤
  - One which if *a* is related to *b* and *b* is related to *c* then *a* is related to *c* for all (*a*,*b*), (*b*,*c*) and (*a*,*c*)
  - Let A = { 1, 2, 3, 4, 5 }



- A digraph is transitive if, for there is an edge from *a* to *c* when there is an edge from *a* to *b* and from *b* to *c*

Exercises

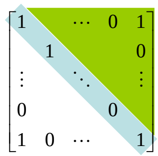


Which of the graphs are reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive

	23	24	25	26	27	28
Reflexive		Y		Y		Y
Irreflexive	Y		Y			
Symmetric					Y	Y
Asymmetric			Y			
Anti-symmetric		Y	Y			
Transitive						Y

How many symmetric relations are there on a set with *n* elements?

- Consider the matrix representing symmetric relation R on a set with *n* elements:
- The center diagonal can have any values
- Once the “upper” triangle is determined, the “lower” triangle must be the transposed version of the “upper” one
- How many ways are there to fill in the center diagonal and the upper triangle?
- There are n^2 elements in the matrix
- There are *n* elements in the center diagonal
  - Thus, there are 2^n ways to fill in 0’s and 1’s in the diagonal
- Thus, there are $(n^2-n)/2$ elements in each triangle
  - Thus, there are $2^{(n^2-n)/2}$ ways to fill in 0’s and 1’s in the triangle
- Answer: there are $2^n * 2^{(n^2-n)/2} = 2^{(n^2+n)/2}$ possible symmetric relations on a set with *n* elements



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