

Fuzzy Sets and Fuzzy Logic

Chapter 14.7
Russell & Norvig

Fuzzy Sets

- Rules of thumb frequently stated in “fuzzy” linguistic terms.

John is *tall*.

If someone is *tall* and *well-built* then his basketball skill is good.

Fuzzy Sets

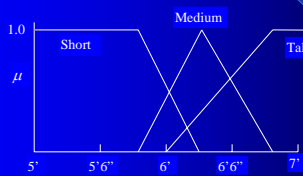
$0 \leq \mu_S(x) \leq 1$ ----- $\mu_S(x)$ (or $\mu(S, x)$) is the degree of membership of x in set S

$\mu_S(x) = 0$ x is not at all in S

$\mu_S(x) = 1$ x is fully in S .

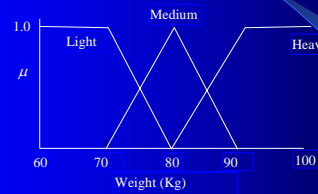
If $\mu_S(x) = 0$ or 1 , then the set S is **crisp**.

Example: Short, Medium height and Tall



Note: $\mu_{Short}(x) + \mu_{Medium}(x) + \mu_{Tall}(x) \neq 1$.

Example: Light, Medium weight and Heavy



Someone who is 6' 1" is simultaneously short, of medium height and tall in various degrees which don't add to 1.

Fuzzy subsets: $S \subseteq T$ if $\mu_S(x) \leq \mu_T(x)$ for all x .

Fuzzy Facts

- Each fuzzy fact is a declaration of the degree of membership in a fuzzy set.

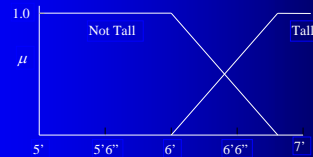
John is *tall* (degree 0.2).
 John is of *medium* height (degree 0.6).
 John is *short* (degree 0.3).
 John is *well-built* (degree 0.7).
 John is *weak* (degree 0.40).

Generally, fuzzy membership functions are defined in terms of numerical values of an underlying **crisp** attribute.
 For example: *Short*, *Medium* and *Tall* in terms of height.
Weak and *Well-built* in terms of muscle mass.

Logical Connectives in Fuzzy Logic

Negation: $\mu(\neg S)(x) = 1 - \mu S(x)$.

The set $\neg S$ is the **complement** of the set S .

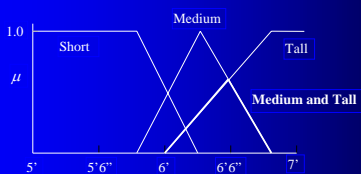


Logical Connectives in Fuzzy Logic

- Conjunction:**

$$\mu(S \wedge T)(x) = \min(\mu S(x), \mu T(x))$$

- Example: *Medium and Tall*

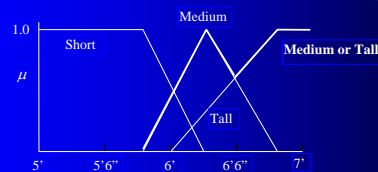


Note: No one is a full member of this set.

Logical Connectives in Fuzzy Logic

- A fuzzy set is normalised when it is possible for someone to be a full member of the set ($\mu = 1$).

Union: $\mu(S \vee T)(x) = \max(\mu S(x), \mu T(x))$.



Remark: Unlike probabilities, fuzzy membership function for negation, conjunction and disjunction are easily calculated.

Fuzzy Rules

If height is *short* then weight is *light*.

Rule 1

If height is *medium* then weight is *medium*.

Rule 2

If height is *tall* then weight is *heavy*.

Rule 3

Problem: Given

- (a) membership functions for short, medium-height, tall, light, medium-weight and heavy;
- (b) the above three fuzzy rules,
- (c) the fact that John's height is 6'1",
Estimate John's weight.

Fuzzy Rules

Problem: Given

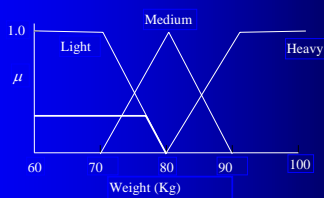
- (a) membership functions for short, medium-height, tall, light, medium-weight and heavy;
- (b) the above three fuzzy rules,
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Estimate John's weight.

Solution

- (1) From John's height we know that
John is *short* (degree 0.3)
John is of *medium* height (degree 0.6).
John is *tall* (degree 0.2).
- (2) (a) Each rule produces a fuzzy set as output by truncating the consequent membership function at the value of the antecedent membership.

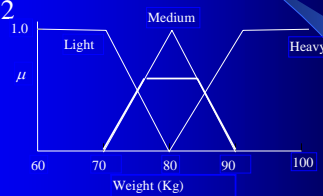
Fuzzy Rules

● Rule 1



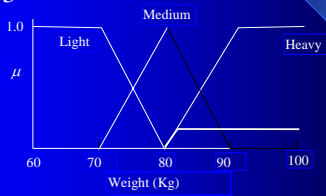
Fuzzy Rules

● Rule 2



Fuzzy Rules

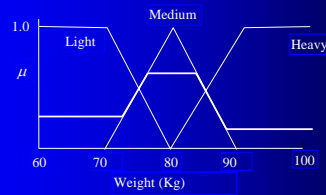
- Rule 3



Fuzzy Rules

(b) The cumulative fuzzy output is obtained by **ORing** the output from each rule.

Cumulative fuzzy output (Weight at 6'1")



Fuzzy Rules

(c) De-fuzzification To obtain a numerical estimate, we need to de-fuzzify the output.

Choose the middle of the range where the truth value is maximum.

(This is one possibility.)

John's Weight = 80 Kgs.

Summary

- Advantages of fuzzy logic
 - Allows the use of vague linguistic terms in the rules.
- Disadvantages of fuzzy logic
 - Difficult to estimate membership function
 - There are many ways of interpreting fuzzy rules, combining the outputs of several fuzzy rules and de-fuzzifying the output.