

22C:145 ARTIFICIAL INTELLIGENCE I
Midterm I (close book and notes)

1. (20 points) Show formally the following propositional sentence is not satisfiable:

$$\neg(((p \wedge q) \rightarrow r) \Leftrightarrow ((p \rightarrow r) \vee (q \rightarrow r)))$$

2. (40 points) Consider the water jug problem: You have two jugs, a 3-gal. and a 4-gal., initially empty. The following operations are considered in the given order when expanding a node:
- (a) If the 3-gal jug is not empty and the other jug is not full, pour water from the 3-gal. jug to the 4-gal jug until one is full or one is empty.
 - (b) If the 4-gal jug is not empty and the other jug is not full, pour water from the 4-gal. jug to the 3-gal jug until one is full or one is empty.
 - (c) Fill the 3-gal. jug with tap water if it's not full.
 - (d) Fill the 4-gal. jug with tap water if it's not full.
 - (e) Empty the 3-gal jug.
 - (f) Empty the 4-gal jug.

The goal is to have 2 gallons of water in the 4-gal jug.

For each search method listed below, please draw a search tree until either the tree has 12 distinct nodes or a solution is found. Note that a node may be generated more than once but there are no repeated nodes in any path from the root. Please mark the nodes with a number to show the time they generated.

- (a) **Depth-First**
 - (b) **Breadth-First**
 - (c) **A*** using the heuristic function that returns the absolute value of the gallons of water in the 4-gal jug minus 2. Each operation has cost 1.
3. (40 points) One version of “Last One Loses” game starts with a stack of n pennies. Two players, A and B, alternate moves. In each move, a player may remove up to $\lfloor k/2 \rfloor$ pennies from the stack, where k is the number of pennies in the current stack, but he must remove at least 1. The player who left the stack with one penny will win.

Player A will use the MINIMAX procedure to decide his move. The function he uses to compute the static value of a node is as follows: Suppose the current stack has k pennies. If it is A's turn, the value will be $f(\lfloor k/2 \rfloor)$; otherwise, the value will be $-f(\lfloor k/2 \rfloor)$, where $f(1) = -100$; $f(x) = 20 - x$ if $x > 1$ and x is odd; $f(x) = 10 - x$ if x is even.

- (a) Suppose the current stack has 8 pennies and it is Player A's turn. Please draw a complete MINIMAX search tree with 3 steps of look-ahead and decide the right move for Player A.
- (b) In (a), if the Alpha-Beta pruning is used and the children of a given node are ordered by the decreasing order of the number of remaining pennies, how many calls to the function f can be saved?
- (c) In (b), if the increasing order is used instead of the decreasing order, how many calls to the function f can be saved?