Midterm I. (22C:135, Fall 2005)

Close books and notes, except one sheet of notes
Total points = 100

1. (30) Let $L = (01)^{+}0(10)^{*}0^{*}$.

   (a) (15) Provide a right-linear grammer for L.
   Note: $(01)^{+} = (01)^{*}(01)$.
   Sample solution:

$$\begin{array}{lcl} S & \rightarrow & 01S \mid 010A \\ A & \rightarrow & 10A \mid B \\ B & \rightarrow & 0B \mid \epsilon \end{array}$$

   (b) (15) Let h be a homomorphism such that $h(0) = ab$ and $h(1) = a$. Please provide an NFA to accept $h(L)$.
   Sample solution: NFA $M = (S, \{a, b\}, \delta, 1, F)$, where $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $F = \{9\}$, and δ is defined by the following table (e.g., $\delta(1, \epsilon) = \emptyset$, $\delta(1, a) = \{2\}$):

δ	1	2	3	4	5	6	7	8	9	10
ϵ						$\{9\}$				
a	$\{2\}$		$\{1, 4\}$	$\{5\}$		$\{7\}$	$\{8\}$		$\{10\}$	
b		$\{3\}$			$\{6\}$			$\{6\}$		$\{9\}$

2. (35) Given an NFA $M = (S, \Sigma, \delta, s_0, F)$, where $S = \{s_0, s_1, s_2, s_3\}$, $\Sigma = \{0, 1\}$, $F = \{s_2\}$ and δ is:

δ	0	1
s_0	$\{s_1, s_3\}$	$\emptyset$
s_1	$\{s_0, s_2\}$	$\{s_1\}$
s_2	$\{s_3\}$	$\{s_0\}$
s_3	$\emptyset$	$\{s_2\}$

   (a) (20) Converting M into an equivalent DFA.
   Sample solution: An equivalent DFA is given below: $D = (S', \{0, 1\}, \delta', 1, F')$, where

$S' = \{1,2,3,4,5,6,7,8,9,10\}$, $F' = \{4,5,6,9,10\}$, and δ' is:

	δ'	0	1
1	$\{s_0\}$	$\{s_1, s_3\}$	$\emptyset$
2	$\emptyset$	$\emptyset$	$\emptyset$
3	$\{s_1, s_3\}$	$\{s_0, s_2\}$	$\{s_1, s_2\}$
4	$\{s_0, s_2\}$	$\{s_1, s_3\}$	$\{s_0\}$
5	$\{s_1, s_2\}$	$\{s_0, s_2, s_3\}$	$\{s_0, s_1\}$
6	$\{s_0, s_2, s_3\}$	$\{s_1, s_3\}$	$\{s_0, s_2\}$
7	$\{s_0, s_1\}$	$\{s_0, s_1, s_2, s_3\}$	$\{s_1\}$
8	$\{s_1\}$	$\{s_0, s_2\}$	$\{s_1\}$
9	$\{s_0, s_1, s_2, s_3\}$	$\{s_0, s_1, s_2, s_3\}$	$\{s_0, s_1, s_2\}$
10	$\{s_0, s_1, s_2\}$	$\{s_0, s_1, s_2, s_3\}$	$\{s_0, s_1\}$

(b) (15) Decide if the DFA obtained in (a) is minimal with justification.
Sample solution: The above DFA is minimal because no two states are equivalent. To show it, we compute the equivalent classes of states as follows:

$$\begin{aligned}\Pi_1 &= \{S' - F', F'\}\\ \Pi_2 &= \{[1,2],[3],[7,8],[4],[5,10],[6],[9]\}\\ \Pi_3 &= \{[1],[2],[3],[7],[8],[4],[5],[10],[6],[9]\}\end{aligned}$$

Since each class contains a single state in Π_3, no two states are 3-equivalent, thus no two states are equivalent in general.

3. (35) Decide if the following languages are regular. If yes, provide one of its regular representations (i.e., DFA, NFA, regular grammar, or regular expression). If not, prove it by the pumping lemma.

(a) (15) $L_1 = \{0^{3x}1^{2y} \mid x, y \geq 0\}$.
Sample solution: L_1 is regular since it can be represented by the regular expression $(000)^*(11)^*$.

(b) (20) $L_2 = \{0^{3x}1^{2y} \mid x \geq y \geq 0\}$.
Sample solution: L_2 is not regular and we can prove this by Pumping Lemma. Suppose L_2 is regular, then there exists a pumping length p. Take $s = 0^{3p}1^{2p} \in L_2$. Since $|s| \geq p$, by the pumping lemma, $s = xyz$ for some x, y, z such that $|xy| \leq p$, $|y| > 0$, and $xy^iz \in L_2$ for any $i \geq 0$. Because $|xy| \leq p$, y must be 0^h for some $h > 0$. However, $xz = 0^{3p-h}1^{2p}$ cannot be in L_2. So L_2 cannot be regular.