2.45 Show that $A_2 = \{wtx^R \mid w, t \in (0+1)^*, |w| = |t|\}$ is not context-free.

Proof. Suppose A_2 is CF and let p be the pumping length. We pick $s = 1^{2p}0^p1^{3p} \in A_2$. By the pumping lemma, $s = uvxyz$ and $uv^ixy^iz \in A_2$ for any i.

There are two cases for the position of vxy:

1. vxy is entirely in the last two thirds of s. Then uv^2xy^2z contains 0's in its first third but not in its last third and so is not in A_2.

2. vxy intersects with the first third of s. Because $|vxy| \leq p$, vxy does not intersects with the second half of s. There are several subcases:

   (a) v contains 0's and possibly 1's. Then uv^2xy^2z contains 0's in its first third but not in in its last third and is not in A_2.

   (b) v is empty and y contains 0's and possibly 1's. Then again uv^2xy^2z contains 0's in its first third but not in in its last third and is not in A_2.

   (c) v or y contain only 1's. Then again $uv^0xy^0z = uxz$ contains 0's in its first third but not in in its last third and is not in A_2.

In all cases, uv^ixy^iz is not in A_2 for some i. So A_2 cannot be context-free