

22C:131 Final Exam

Close books and notes, except three sheets of notes

Total points = 100

1. (30) Decide if the following language L_1 is decidable. If yes, provide an algorithm to solve the problem; if not, prove it formally (you may assume any undecidable language discussed in the class).

$$L_1 = \{\langle M, s \rangle \mid M \text{ is a TM, } s \text{ is a state to be used by } M\}$$

2. (30) Let $SET\text{-}SPLITTING = \{\langle S, C \rangle \mid S \text{ is a finite set and } C = \{C_1, \dots, C_k\} \text{ is a collection of subsets of } S, \text{ for some } k > 0, \text{ such that elements of } S \text{ can be colored } \textit{red} \text{ or } \textit{blue} \text{ so that no } C_i \text{ has all its elements colored with the same color.}\}$ Show that (a) $SET\text{-}SPLITTING$ is in NP; (b) $3SAT \leq_P SET\text{-}SPLITTING$.
3. (20) Given a set $X = \{x_1, x_2, \dots, x_n\}$ of n real numbers, we have two computation jobs to perform on X :
 - (a) sort X ;
 - (b) compute $\sum_{1 \leq i, j \leq n} \sqrt{x_i x_j} / n^2$.

Show that there exists a function $f(n)$ such that one job is in $\text{TIME}(f(n))$ but the other job is not in $\text{TIME}(f(n))$.

4. (20) Let M be a probabilistic polynomial time turing machine and let C be a language where, for some fixed $0 < \epsilon_1 < \epsilon_2 < 1$,
 - (a) $w \notin C$ implies $\Pr[M \text{ accepts } w] \leq \epsilon_1$, and
 - (b) $w \in C$ implies $\Pr[M \text{ accepts } w] \geq \epsilon_2$.

Show that $C \in \text{BPP}$.