Home work 6 sample solution

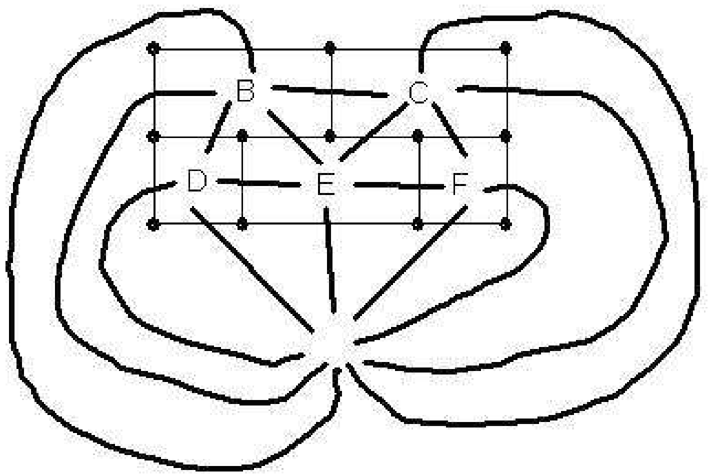
22C:034 Spring 2004

Q1:
One of the feasible paths:
$< 8, 0, 0 > \rightarrow < 3, 5, 0 > \rightarrow < 3, 2, 3 > \rightarrow < 6, 2, 0 > \rightarrow < 6, 0, 2 > \rightarrow < 1, 5, 2 > \rightarrow < 1, 4, 3 > \rightarrow < 4, 4, 0 >$

Q2:
Label each of the six regions in the given figure as indicated below and take these as nodes of a related graph. Join two of these nodes by an edge if they are separated by one of the line segments in question. The result is the connected graph shown below, and an Euler path for this graph would provide the requested line for the original graph. Inspection of this new graph reveals:

degree(A) = 9
degree(B) = 5
degree(C) = 5
degree(D) = 4
degree(E) = 5
degree(F) = 4

Since there are not exactly two nodes of odd degree, by the Euler path theorem, there is no Euler path in the derived graph and hence no single line for the original.



Q3:
Proof:

$\Rightarrow$) By definition of tree, a tree is an undirected simple graph that is connected and has no cycles. So if G is a tree, it has no non-null simple cycles. Suppose we add an edge u, v, and we get G'. Since $G(V, E)$ is connected, so in G,there is a simple path from u to v, suppose it is $p1 = (u, v1, ..., v_k, v)$,then $(u, v1, ..., v_k, v, u)$ is a simple cycle. So adding any new edge creates a simple cycle.

$\Leftarrow$) We'll show first that G has no cycles, then G is connected.

First,G has no cycles. Obvious from the assumption that G has no non-null simple cycles.

Second, G is connected, i.e. any vertex pair (u, v), v is reachable from u. If edge $u, v \in E$ then of course v is reachable from u. Otherwise, add a new edge u,v. By assumption, adding any new edge creates a simple cycle. Because G has no cycle, so edge u,v is in the simple cycle. Suppose the simple cycle is $(u, v_1, v_2, ...v_k, v, u)$, then $(u, v_1, v_2, ...v_k, v)$ is a simple path from u to v in G. So v is reachable from u in G.

So proved.

Q4: See the trace in Figure 1 in the next page.

Q5:

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 2 & 1 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 1 & 2 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 3 & 2 & 3 \\ 0 & 4 & 1 & 3 \\ 0 & 3 & 2 & 3 \\ 0 & 1 & 2 & 2 \end{bmatrix}$$

According to A, There is only 1 path of length 1 from v_1 to v_4, i.e. (v_1, v_4). This is also the only elementary path of length 1 from v_1 to v_4.

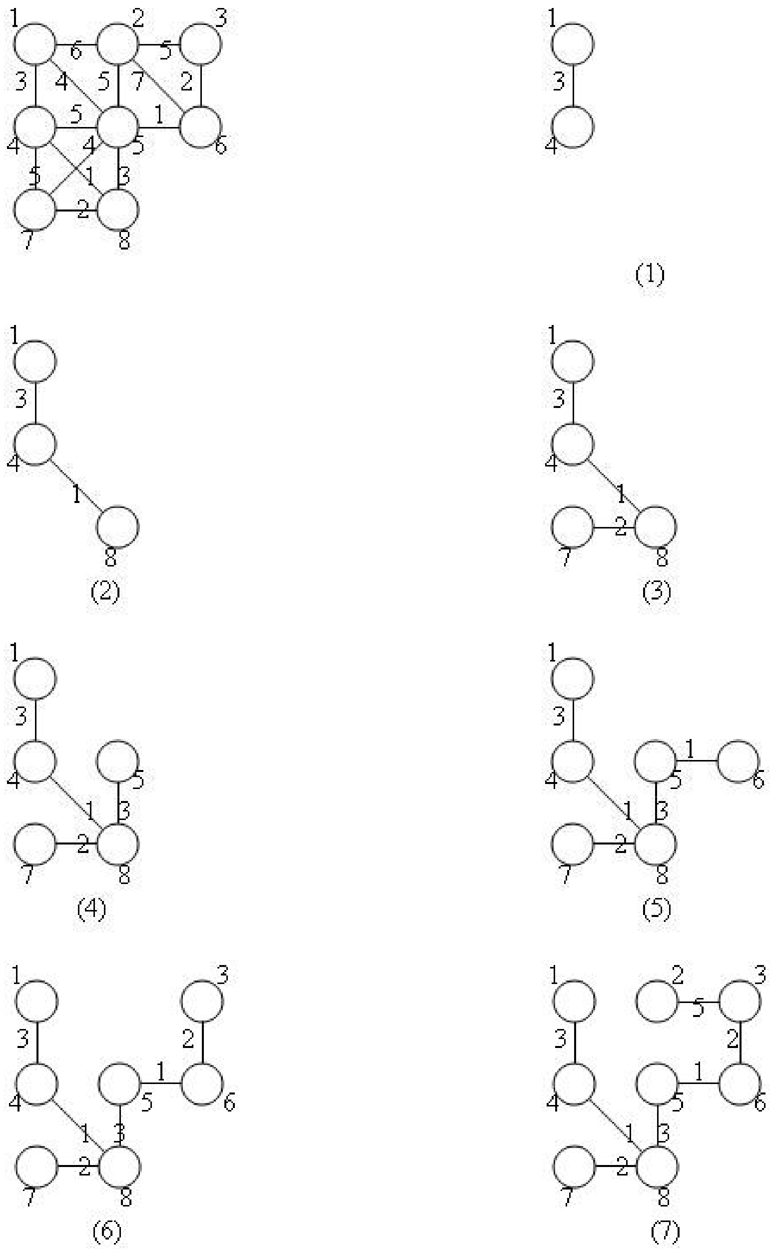


Figure 1: trace of Prim's method

According to A^2, There is only 1 path of length 2 from v_1 to v_4, i.e. (v_1, v_2, v_4). This is also the only elementary path of length 2 from v_1 to v_4.

According to A^4, There are totally 3 paths of length 4 from v_1 to v_4. These paths are:

(1)$p_1 = (v_1, v_2, v_4, v_2, v_4)$.

(2)$p_2 = (v_1, v_2, v_3, v_2, v_4)$.

(3)$p_3 = (v_1, v_4, v_2, v_3, v_4)$.

Among the three paths, p_2, p_3 are simple paths. So we can see there is a simple path of length 4 from v_1 to v_4.