Fall 2005 22C:111

Homework1 Solution

Problem 1:

(a) $\{\varepsilon, bb, bbbb\}$

(b) {zero or even number of 'b's}

(c) {odd number of 'b's, or aba followed by zero or even number of 'b's}

(d) $= S^*$

(e) $= S^*$

Problem 2:

(a) $\{a\}^*$

(b) S

(c) R

(d) $\{a\}^*$

Problem 3:

(a) $(aaa)^*|(aaaaa)^*$

(b) $(aaa^*b^*)|(bbb^*a^*)$

Let

$L_{11} =$ {Strings that begin with 'aa' and have no 'a' following a 'b'}

$L_{12} =$ {Strings that begin with 'bb' and have no 'b' following an 'a'}

$L_1 =$ {Strings that either (i) begin with 'aa' and have no 'a' following a 'b', or (ii) begin with 'bb' and have no 'b' following an 'a' }

$L_{21} = L(aaa^*b^*)$, which denotes the set of strings described by the regular expression

$L_{22} = L(bbb^*a^*)$

$L_2 = L((aaa^*b^*)|(bbb^*a^*))$

Then $L_1 = L_{11} \cup L_{12}$, $L_2 = L_{21} \cup L_{22}$

We'll show that $L_{11} = L_{21}$.

Proof: If $s \in L_{11}$, then s begins with 'aa' and have no 'a' following a 'b'. Then $s = aa \cdot t$, where t is a string that has no 'a' following a 'b'. So t can be described by a^*b^*. So s can be described by aaa^*b^*, then $s \in L_{21}$, then $L_{11} \subseteq L_{21}$.

If $s \in L_{21}$, then obviously s begins with 'aa' and have no 'a' following a 'b'. Then $s \in L_{11}$, $L_{21} \subseteq L_{11}$.

Therefore, we have $L_{11} = L_{21}$. Similarly we can show that $L_{12} = L_{22}$, then we have $L_1 = L_2$.

(c) $(aa|a)(aaa)^*$

Problem 4:

(a) Yes

X$\Rightarrow a^3Xa$
$\Rightarrow a^5Xa^3$
$\Rightarrow a^7Xa^5$
$\Rightarrow a^7ba^5$

(b) No.Because the total number of 'a's in the string in L(X) should be multiple of 4.

(c) Yes

X$\Rightarrow a^3Xa$
$\Rightarrow a^6Xa^2$
$\Rightarrow a^8Xa^4$
$\Rightarrow a^8ba^4$

Problem 5:

(a) Yes

X$\Rightarrow XaXbX$
$\Rightarrow aXbX$
$\Rightarrow aXaXbXbX$
$\Rightarrow aaXbXbX$
$\Rightarrow aabXbX$
$\Rightarrow aabbX$
$\Rightarrow aabbXaXbX$
$\Rightarrow aabbaXbX$
$\Rightarrow aabbabX$
$\Rightarrow aabbab$

(b) No. Because number of 'a's $\neq$ number of 'b's.

(c) No. Because for any string in L(X), at any place in the string, the number of 'a's before should $\geq$ the number of 'b's before. (This statement can be proved by induction.) But in this string as it's marked, $abb|aab$, number of 'b's = 2, number of 'a's = 1 at the mark.

(d) Yes

X$\Rightarrow XaXbX$
$\Rightarrow aXbX$
$\Rightarrow aXaXbXbX$
$\Rightarrow aaXbXbX$
$\Rightarrow aaXaXbXbXbX$
$\Rightarrow aaaXbxbXbX$
$\Rightarrow aaabXbXbX$
$\Rightarrow aaabbXbX$
$\Rightarrow aaabbbX$
$\Rightarrow aaabbb$