22C:111 -- Fall 2005

Homework 10
sample solutions

Problem 1

(a) x-y+1 -- this is greater than 0 when $x \geq y$

(b) 1-x*x -- this is greater than 0 only when $x=0$

(c) x*x -- this is greater than 0 whenever $x \neq 0$

Executable versions of solutions for problems 2 &3 are in the class directory.

Problem 4.

Step 1: discover the loop invariant

The loop invariant in conjunction with the negation of the loop guard must imply the post-condition. So the "difference" between these conditions is a general guide for what is needed for the loop invariant. In this case the loop invariant ≡ $n>(sqrt-1)^2$.

Step 2: prove invariant true at first arrival -- {n≥1} sqrt:= 1 {loop invariant}

by the Axiom of assignment

```
{ n>0 }
  sqrt:= 1
{ n>(sqrt-1)^2 }
```

Step 3: prove this assertion is "invariant" -- still true after the execution of the loop body, given the loop guard.

3A. by the Axiom of Assignment

```
{ n>sqrt^2 }
  sqrt:= sqrt+1
{ n>(sqrt-1)^2 }
```

3B. by Strengthening the pre-condition in Step 3A, Step 3 is proven since

$n>sqrt^2 \land n>(sqrt-1)^2 \Rightarrow n>sqrt^2$

Step 4: prove the While post-condition

by the While rule on Step 3

```
{loop invariant}
   while n-sqrt*sqrt do
     sqrt:= sqrt+1
     od
{ loop invariant ∧ n≤sqrt^2 }
```

Step 5: prove the program

by the Sequential execution rule on Steps 2 and 4, at the conclusion of the program the post-condition { loop invariant $\land n \leq sqrt^2$ } = { $n>(sqrt-1)^2 \land n \leq sqrt^2$ } is proven.