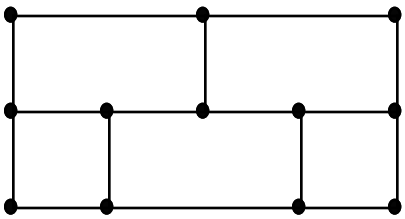
Homework VI

1. [10 points]
Two persons have an 8 gallon jug of wine, plus empty 5 gallon and 3 gallon jugs. They wish to divide the wine evenly between them using only the equipment immediately at hand. Since none of the jugs have any measurement marks on them, the available steps are to pour one jug into another, either emptying the first or filling the second. The problem is to find a series of steps to achieve the desired end.

As a digraph, nodes can be regarded as triples of natural numbers <i, j, k> representing the contents of the three jugs, where $0 \le i \le 8$, $0 \le j \le 5$, $0 \le k \le 3$, and $i+j+k = 8$. Edges are determined by the allowed transfers, e.g., <3,3,2> to <6,0,3> and <2,4,2> to <2,5,1>. The problem seeks a path from <8,0,0> to <4,4,0> -- find such a path.

2. [15 points]
In the figure below, can a single continuous line be drawn that crosses each of the 16 line segments (between the dots) exactly once? Justify your answer.



3. [20 points]
Prove that graph G is a tree if and only if G has no non-null simple cycles and adding any new edge creates a simple cycle.

4. [15 points]
Problem 7(b), p. 421 of our text

5. [15 points]
Problem 1, p. 390 of our text