# Sets, Relations, Functions & Sequences

Z uses logic to describe models using the mathematical entities listed above. The text coverage of these topics is in:

- chapter 3
- chapter 5, section 1
- chapter 6, sections 3 & 4
- chapter 7

Z reference material is in chapter 21, or see the Z Reference Manual by Spivey (on-line).

Z is a “typed” notation — there is normally a variety of kinds of objects (e.g., integers, functions, and sequences) simultaneously under consideration, and variables are constrained to denote only objects from a designated domain. Often the domain is “generic” — left to be determined as needed. Z uses the notation [X,Y] to denote that X and Y are generic domains.

A <u>set</u> is an aggregation of objects — order is immaterial, and objects may not be repeated. The defining property is <u>membership</u> — for item x in set X, this is written $x \in X$. Two sets X and Y are <u>equal</u>, written X=Y, if they contain exactly the same members.

Sets are described either by <u>enumeration</u> or by <u>comprehension</u>. With enumeration, we explicitly record each of the items of the set. For example, {2,3,5,7} is the set of the first four primes. Of course, {5,3,7,2} is the same set, and we write {2,3,5,7} = {5,3,7,2}.

Set descriptions can also be based on other known sets using comprehension. This takes the form

{x:X | “condition on x” • “term in x”}

and describes all items such that for each x in set X, if the condition is true, then the resulting term value belongs to the comprehension. For instance, {x:{2,3,5,7} | x<6 • 3*x} = {6,9,15}.

Finally, a key set former is the power set operator, $\mathbb{P}$. For a set X, $\mathbb{P}$ X denotes the set of all subsets of X. For instance,

$\mathbb{P}$ {2,3,5} = {∅, {2}, {3}, {5}, {2,3}, {2,5}, {3,5}, {2,3,5}}. In general, if a set contains N items, then the power set has $2^N$ items. Of course, the power set operator can be applied to infinite as well as finite sets.