22C : 044 Algorithms
Spring 2002
Homework 6
Due on April 18

1. Given a set of n distinct positive integers $S = \{s_1, \ldots, s_n\}$, and a target integer t, determine if there is a subset $S' \subseteq S$ of S such that the elements in S' add up to t. For example within $S = \{1, 2, 5, 9, 10\}$ there is a subset which adds up to $t = 22$ but not $t = 23$. We want to develop a dynamic programming solution to this problem.

   We say that a set Q of positive integers *achieves* an integer w if there is a subset $Q' \subseteq Q$ such that the elements of Q' add up to w. By convention, we will assume that any set Q achieves 0. Note that we want to determine if the set S achieves t. For $1 \leq i \leq n$, let $S_i = \{s_1, \ldots, s_i\}$ be the set obtained by taking the first i elements of S. Note that $S = S_n$. Let $S_0 = \emptyset$.

   (a) Prove that for $1 \leq i \leq n$ and any $w > 0$, S_i achieves w if and only if either S_{i-1} achieves w or S_{i-1} achieves $w - s_i$.

   (b) Use this relation to give a dynamic programming algorithm that as a by-product determines if S achieves t. The algorithm should run in $O(nt)$ time. Hint: First determine the numbers that S_1 achieves, then the numbers that S_2 achieves, ... , and finally the numbers that S_n achieves.

2. Solve Problem 15-4 from the textbook. To simplify the book-keeping, assume that the tree is binary and is represented like a binary tree. (The problem statement in the book allows a node in the tree to have more than two children.) To solve this problem, you have to first relate the optimal solution at a node of the tree to the optimal solutions at its children and grandchildren.