University of Iowa
Department of Computer Science

Midterm Examination II

22c:196 Logic in Computer Science
Spring 2007

Please read these directions but wait for the signal to begin working. Be sure to follow all directions as to the required form of your answer.

Print your name neatly in the space provided below; print your name at the upper right corner of every page.

Name: **Key**

This booklet should have 7 pages (including this one). If it does not, report this to the proctor immediately.

Make sure you efficiently and do not get stuck too long on any one problem.

Do not use this booklet for scratch work. Write your final answer in the space provided and use the back of the pages only if you need more space.

Question	Your Score	Possible
1		21
2		18
3		12
4		08
5		41
extra		(06)
Total		100

1 English to LTL (21 points)

For each of the following statements English provide the most accurate translation in LTL.

1. If p ever occurs, q will eventually occur too.

   $G\,(p \rightarrow F\,q)$

   Also accepted (but not equivalent): $F\,p \rightarrow Fq$

2. At some point, the value of p will not change anymore.

   $F\,(G\,p \vee G\,\neg p)$

3. p will hold eventually but not now.

   $\neg p \wedge F\,p$

4. p and q never hold at the same time.

   $G\,\neg(p \wedge q)$

5. Once p occurs, q holds until s occurs.

   $G\,(p \rightarrow (q\,U\,s))$

6. Once p becomes true, q stays the same as long as p holds.

   $G\,(p \wedge q \rightarrow (q\,U\,\neg p)) \wedge G\,(p \wedge \neg q \rightarrow (\neg q\,U\,\neg p))$

7. p holds in the current state and in every other state after that.[1]

   $p \wedge G\,(p \rightarrow X\,X\,p)$

   Also accepted (but stronger): $p \wedge G\,(p \leftrightarrow X\,X\,p)$

[1] That is, p holds for sure in the first state, in the third, in the fifth, and so on.

2 English to CTL (18 points)

For each of the following statements English provide the most accurate translation in CTL.

1. No matter what, at some point p will become permanently true.
   $AF\,AG\,p$

2. From any state it is possible to reach a state that satisfies p but not q.
   $AG\,EF(p \wedge \neg q)$

3. Any state in which p holds is unreachable.
   $\neg EF\,p$

4. It is always possible to make q true once p is true.
   $AG\,(p \rightarrow EFq)$

5. No matter what, p will hold until q or r holds.
   $A\,[p\,U\,(q \vee r)]$

6. There is a path in which p and q occur one right after the other, without overlapping.
   $EF\,((p \wedge \neg q \wedge EX\,(\neg p \wedge q)) \vee (\neg p \wedge q \wedge EX\,(p \wedge \neg q)))$

3 LTL Semantics (6 + 6)

Consider the formulas

$$\varphi_1 = F\,p \rightarrow F\,q$$
$$\varphi_2 = G\,(p \rightarrow F\,q)$$

1. Provide a model and and state s_0 such that $\mathcal{M}, s_0 \models \varphi_1 \wedge \varphi_2$, if such a model exists. Otherwise, argue why no such model exists.

   Consider the model $\mathcal{M} = (S, \rightarrow, L)$ where:

   $$S = \{s_0,\, s_1\}$$
   $$\rightarrow\; = \{(s_0, s_1),\, (s_1, s_1)\}$$
   $$L = \{s_0 \mapsto \{p\},\, s_1 \mapsto \{q\}\}$$

   which has just one path originating from s_0, namely $s_0\,(s_1)^\omega$.

   That path satisfies φ_1 because it satisfies both $F\,p$ and $F\,q$, and it satisfies φ_2 because q holds in a future state of s_0, the only state that satisfies p.

   It follows that $\mathcal{M}, s_0 \models \varphi_1 \wedge \varphi_2$

2. Provide a model and and state s_0 such that $\mathcal{M}, s_0 \models \varphi_1$ and $\mathcal{M}, s_0 \not\models \varphi_2$, if such a model exists. Otherwise, argue why no such model exists.

   Consider the model $\mathcal{M} = (S, \rightarrow, L)$ where:

   $$S = \{s_0,\, s_1\}$$
   $$\rightarrow\; = \{(s_0, s_1),\, (s_1, s_1)\}$$
   $$L = \{s_0 \mapsto \{q\},\, s_1 \mapsto \{p\}\}$$

   which has just one path originating from s_0, namely $s_0\,(s_1)^\omega$.

   That path satisfies φ_1 because it satisfies both $F\,p$ and $F\,q$. However, it does not satisfy φ_2 because p holds in s_1 but q holds in no future state of s_1.

4 CTL Syntax (4 + 4 points)

Draw the parse tree of each of the following CTL formulas.

1. $AG\,(p \rightarrow EG\,q)$

2. $A\,[E[p\,U\,q]\,U\;EG\,q]$

5 CTL Semantics (2 + 9 + 30)

1. Draw the following model $\mathcal{M} = (S, \rightarrow, L)$ where:

$$\begin{aligned} S &= \{s_0,\, s_1,\, s_2,\, s_3\} \\ \rightarrow &= \{(s_0, s_1),\, (s_0, s_3),\, (s_1, s_1),\, (s_1, s_2),\, (s_2, s_0),\, (s_2, s_3),\, (s_3, s_0)\} \\ L &= \{s_0 \mapsto \{p, q\},\, s_1 \mapsto \{r\},\, s_2 \mapsto \{p, t\},\, s_3 \mapsto \{q, r\}\} \end{aligned}$$

2. Provide a list of ω-regular expressions (as seen in Homework 5) that capture all the possible paths in $\mathcal{M}$ that start from s_0 *and do not contain* the transition $s_2 \rightarrow s_0$.

3. Write Y or N in the table below depending on whether the given state satisfies the given formula in the above model $\mathcal{M}$ or not.

   For this questions you get 3 points for each correct answer, -3 points for each incorrect answer, and 0 points for each blank one, with any total score of less than -9 points adjusted to -9 points.

	s_0	s_2
$AF\,q$		
$AG\,EF\,(p \vee r)$		
$EX\,EX\,r$		
$AG\,AF\,q$		
$A\,[(r \vee q)\,U\,q]$		

- $\mathcal{M}, s_0 \models AF\,q$ trivially because q holds in s_0 already.
- $\mathcal{M}, s_2 \models AF\,q$ because all paths originating from s_2 go through s_0 or s_3 where q holds.
- $\mathcal{M}, s_0 \models AG\,EF\,(p \vee r)$ because, for instance, r holds in s_1 and s_1 is reachable from any state reachable from s_0.
- $\mathcal{M}, s_2 \models AG\,EF\,(p \vee r)$ because, for instance, r holds in s_1 and s_1 is reachable from any state reachable from s_2.
- $\mathcal{M}, s_0 \models EX\,EX\,r$ because r holds in s_1 and s_1 is reachable in two steps from s_0 along the path $s_0\,(s_1)^\omega$.
- $\mathcal{M}, s_2 \models EX\,EX\,r$ because r holds in s_1 and s_1 is reachable in two steps from s_2 along the path $s_2\,s_0\,(s_1)^\omega$.
- $\mathcal{M}, s_0 \not\models AG\,AF\,q$ because, for instance, s_1 is reachable from s_0, however, q never holds along the path $(s_1)^\omega$.
- $\mathcal{M}, s_2 \not\models AG\,AF\,q$ because, for instance, s_1 is reachable from s_2, however, q never holds along the path $(s_1)^\omega$.
- $\mathcal{M}, s_0 \models A\,[(r \vee q)\,U\,q]$ trivially because q holds in s_0 already.
- $\mathcal{M}, s_2 \not\models A\,[(r \vee q)\,U\,q]$ trivially because neither q nor r holds in s_2 to start with.

6 Optional, extra credit (6 points)

Let $\mathcal{M} = (S, , \rightarrow, L)$ be a model and recall that, for every subset Y of S,

$$\begin{aligned} \text{pre}_{\exists}(Y) &:= \{s \in S \mid s \rightarrow s' \text{ for some } s' \in Y\} \qquad (1) \\ \text{pre}_{\forall}(Y) &:= \{s \in S \mid s' \in Y \text{ for all } s' \text{ s.t. } s \rightarrow s'\} \qquad (2) \end{aligned}$$

Prove that for every subset Y of S,

$$\text{pre}_{\forall}(Y) = S - \text{pre}_{\exists}(S - Y) \ .$$