22C:44 Homework 6

Due by 5pm on Tuesday, 4/17

Each problem is worth 10 points.

- 1. Problems 12.3-4 (page 232) and 12.4-1 (page 240).
- 2. Consider the hash function

$$h(k, i) = (k \mod m + c \cdot i) \mod m$$

where c is some positive integer. Suppose that we want to use this hash function for open addressing. Characterize values of c that will make the probe sequence

$$h(k,0), h(k,1), \ldots, h(k,m-1)$$

a permutation? Your friend claims that for an appropriately chosen c this hash function is better than linear probing. What do you think?

- 3. Problem 12-4 on Page 242.
- 4. Problem 17-1 (b) on Page 353 with c = 2.

Hint: Mimic the proof of correctness you saw in class when the denominations were quarters, dimes, nickels, and pennies. In particular, let $\{c_1, c_2, \ldots, c_N\}$ be greedy change and let $\{f_1, f_2, \ldots, f_M\}$ be optimal change, with M < N. Here $c_1 \geq c_2 \geq \cdots \geq c_N$ and $d_1 \geq d_2 \geq \cdots \geq d_N$. Consider i such that $c_j = f_j$ for all j < i, and $c_i > f_i$. It is possible to obtain a contradiction by showing that $(f_i + f_{i+1} + \cdots + f_M) < c_i$.

5. Determine if the following "greedy" algorithm is correct for the activity selection problem. Prove your claim. Here A is the given set of activities (intervals).

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\begin{split} &\text{GreedyActivitySelection(A)} \big\{ \\ &\text{S} \leftarrow \emptyset; \\ &\text{while } (\texttt{A} \neq \emptyset) \text{ do } \big\{ \\ &\text{a} \leftarrow \text{ interval that is compatible with most intervals in A}; \\ &\text{S} \leftarrow \texttt{S} \cup \big\{ \texttt{a} \big\}; \\ &\text{Remove from A all intervals incompatible with a}; \\ &\big\} \\ &\text{return S}; \\ \big\} \end{split}
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