Algorithmic version of L^3.

BECK 1991

Given the proof of existence of a certain object via L^3, is it possible to efficiently construct it ?

k-SAT

The problem k-SAT is the following:-

Input:-A set of classes in disjunctive normal form so that each clause has exactly k literals.

Question:-Is there a satisfying truth assignment to the variables?

k-SAT(l)

Input:-Same as k-SAT except that each variable occurs in exactly l clauses.

*Question:-I*s there a satisfying truth assignment to the variables?

Consider k-SAT($2^{k/50}$) e.g. 100-SAT(4)

Claim:-k-SAT($2^{k/50}$) is always satisfiable.

Proof:-

To each variable assign at random the value T or F independently and with equal probability.Let C be a clause.

Prob.[C is not satisfied]=$1/2^k$.

Let A_C =C is not satisfied.

Prob.[$\bigwedge_C A_C$'] $> 0 =>$ there is a truth assignment satisfying all clauses.

What is d for $\{A_C \mid C$ is a clause$\}$?

d$\leq$ k . $2^{k/50}$

so e.p(d+1) = e.($1/2^k$)(k.$2^{k/50}$+ 1) $\leq$1 for all k$\geq$ 4.

Comment:- 2-coloring uniform hypergraphs is another context in which Beck's algo. is described.

Is there an efficient algo. to find a satisfying truth assignment for k-SAT ($2^{k/50}$) ?

We will describe an algorithm that runs in polytime in m(number of clauses) but not in k .

Implication:-We have a poly-time algo. for k=O(1).

Consider for example : MAX-3SAT(6) , which is quite hard to approximate.

ALgorithm:-

Stage 1 :-

-Order the variables in some arbitrary order.

-Process the variables in this order,assigning to each var. the value T or F with equal prob.,except if the var. belongs to a dangerous clause.

A clause C is dangerous if

i) C has k/2 literals that have been assigned a value.

ii) C is not satisfied.

At the end of stage 1 we have some satisfied clauses and some surviving clauses.A surviving clause is a clause in which no literal has been

assigned the value T.

Remark:-

A clause may be surviving because all its unassigned variables participate in dangerous clauses.

Let G be the graph whose vertex set is the set of clauses and whose edge set = { { C_1,C_2 } | C_1 and C_2 share a variable }

Note that degree $_G$ (C) $\leq$k . $2^{k/50}$ = d

Let H = subgraph of G induced by surviving clauses and unassigned variables.

We can show that with high prob. (1 - O(1)) every connected component of H is O(logm) in size.

Lemma:-

With prob.(1 - O(1)),every connected component of H is of size <= Z.log(m) for some fixed constant Z.

Proof:-

Let C_1,C_2,...,C_r be a collection of clauses such that dist$_G$ (C_i , C_j) $\geq$4 for any i$\neq$j

We will now discuss the prob. that C_1,C_2,...,C_r all survive stage 1. C_i survives only because some clause in {C_i}$\cup$ N(C_i) is dangerous.

Prob.[a clause becomes dangerous in stage 1] = $1/2^{k/2}$.

To each C_i one can associate a clause D_i $\in${C_i}$\cup$ N(C_i) that becomes dangerous in stage 1 .

Prob.[all of D_i s are dangerous] = ($1/2^{k/2})^r$.