Homework 2
Due in class on Noveember 13th

1. In class, I presented the following algorithm for computing an MIS on a UBG in doubling metric spaces. Let d_{min} be the smallest distance between a pair of nodes in the graph.

   $r \leftarrow \{\frac{1}{2^\lambda} \mid \lambda$ is an integer, $\frac{1}{2^\lambda} < d_{min} \leq \frac{1}{2^{\lambda-1}}\}$
   $I \leftarrow V$
   while $(r \leq 1)$ **do**{
       $E_I \leftarrow \{\{u, v\} \mid u, v \in I, d(u, v) \leq r\}$
       $G_I \leftarrow (I, E_I)$
       $I \leftarrow MIS(G_I)$
       $r \leftarrow 2 \cdot r$
   }
   return I

   This was derived from a PODC 2005 paper by Kuhn, Moscibroda, and Wattenhofer. I had claimed that this algorithm returns an MIS of an n-vertex input graph $G = (V, E)$ in $O(\log^* n)$ rounds, provided G is a UBG in a doubling metric space. This is not quite correct and as stated the algorithm does not return an MIS. It does return an independent set, but this set may not be maximal.

   Explain with a simple example, why this algorithm does not always return an MIS. Also, suggest a way of "fixing" the algorithm so that it remains deterministic, still runs in $O(\log^* n)$ rounds, and returns an MIS. You are welcome to read the Kuhn-Moscibroda-Wattenhofer paper to get hints.

2. Consider the *caterpillar graph* from the Jia-Rajaraman-Suel (JRS) paper on minimum dominating sets. The JRS paper claims that if "rounded spans" are used (instead of spans) then the simple distributed greedy algorithm due to Liang and Haas (2000, INFOCOM) will compute an $O(1)$-approximation to a minimum dominating set in $O(\log n)$ rounds on this caterpillar graph. Prove or disprove this claim. If you disprove this claim, then suggest a simple modification to the Liang and Haas algorithm that might work.

3. In the Awerbuch, Goldberg, Luby, and Plotkin paper, there is a remark informing us that "it is easy to construct an n-vertex graph G, such that even if we allow the cluster diameter to be as high as $\sqrt{n}$, the maximum degree of the cluster graph is $\Omega(\sqrt{n})$." Provide an example of one such family of graphs.

4. (i) Show that there is a deterministic algorithm that runs in $O(\log^* n)$ rounds and computes an $O(1)$-approximation for the minimum dominating set (MDS) problem on constant-degree graphs.

   (ii) Show that there is a deterministic algorithm that runs in $O(\log^* n)$ rounds and computes an $O(1)$-approximation for the minimum dominating set (MDS) problem on UBGs in doubling metric spaces.

5. Suppose your friend tells you that she knows of a way to deterministically compute a $(10, 20)$-ruling forest with respect to any $V' \subseteq V$ in $O(\log n)$ rounds. Explain the implications of such a result to the world of distributed algorithms.