22C:131 Homework 3

Due: Wednesday, 4/5

Notes: (a) Solve all 5 problems listed below. The problem numbers refer to problems in the second edition of Sipser. (b) It is possible that solutions to some of these problems are available to you via other theory of computation books or on-line lecture notes, etc. If you use any such sources, please acknowledge these in your homework. You will benefit most from the homework, if you sincerely attempt each problem on your own first, before seeking other sources. (c) It is okay to discuss these problems with your classmates. Just make sure that you take no written material away from these discussions.

1. 8.15 (8.15 in the first edition also).

2. 8.19 (8.21 in the first edition).

3. 8.23. This is missing from the first edition, so it is stated below.
   Define $UCYCLE = \{\langle G \rangle \mid G$ is an undirected graph that contains a simple cycle$\}$. Show that $UCYCLE \in L$. (Note: G may be a graph that is not connected.)

4. *Steve's class* **SC** is defined as the class of languages, each of which can be decided by a deterministic TM that runs in polynomial time *and* uses polylogarithmic space. A TM is said to use polylogarithmic space, if it uses $O(\log^k n)$ space, where k is a constant and n is the size of the input. Use Savitch's Theorem to either show that $\mathbf{NL} \subseteq \mathbf{SC}$ or explain why Savitch's Theorem does not apply here.

   Notes: SC was named after Stephen Cook, famous for the Cook-Levin Theorem. For various reasons Steve's class is not as well known as Nick's class, **NC**. **NC** is a complexity class that attempts to categorize problems by virtue of how difficult they are to solve using parallel algorithms.

5. For each of the claims below, write `True`, `False`, or `Unknown`. If you do write `True` or `False`, you need to provide a brief justification. This does not need to be a proof - a proof sketch or a pointer to a theorem - will suffice.

   (a) $\mathbf{L} \subseteq \mathbf{P}$.

   (b) $\mathbf{NL} \subseteq \mathbf{P}$.

   (c) $\mathbf{L}^2 \subseteq \mathbf{NP}$. Here I am using $\mathbf{L}^2$ to denote $SPACE(log^2 n)$.
