Logic in computer Science

Introduction

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Dictionary: Logic

1.
- (a) a science that deals with the principles and criteria of validity of inference and demonstration: the science of the formal principles of reasoning, e.g., *a professor of logic*
- (b) a branch or variety of logic, e.g., *Boolean logic*
- (c) a branch of semiotics (the study of signs and symbols and their use or interpretation), especially: Syntactics
- (d) the formal principles of a branch of knowledge, e.g., *the logic of grammar*

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Dictionary: Logic (cont.)

2.
- (a) a particular mode of reasoning viewed as valid or faulty. e.g., *She spent a long time explaining the situation, but he failed to see her logic.*
- (b) Relevance, Propriety, e.g., *Could not understand the logic of such an action.*
- (c) interrelation or sequence of facts or events when seen as inevitable or predictable, e.g., *By the logic of events, anarchy leads to dictatorship.*
- (d) the arrangement of circuit elements (as in a computer) needed for computation

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Logic is everywhere in CS

- Discrete Math
- Data Structures
- Algorithms
- Artificial Intelligence
- Computation Theory
- …

Logic is also extensively studied in **Mathematics** and **Philosophy**

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Historical view

- Philosophical Logic
  - 500 BC to 19th Century
- Symbolic Logic
  - Mid to late 19th Century
- Mathematical Logic
  - Late 19th to mid 20th Century
- Logic in Computer Science
  - Everlasting …

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Philosophical Logic

- 500 B.C – 19th Century
- Logic dealt with arguments in the natural language used by humans.
- Example
  - Premise: All men are mortal.
  - Premise: Socrates is a man
  - Conclusion: Socrates is mortal.
- What logic is used here?

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Symbolic Logic

- Mid to late 19th Century.
- Attempted to formulate logic in terms of a mathematical language
- Rules of inference were modeled after various laws for manipulating symbolic expressions.

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Method of Symbolic Logic

- Step 1: Use certain symbols to express the abstract form of statements, some of them are assumed (premises) and some of them needed to be derived (conclusions).
- Step 2: Use a certain procedure to figure out whether the conclusion follows from the premises based on their abstract symbolizations.

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Statements in Logic

- A statement is a declaration whose value is either true or false.
- A statement is also called proposition.
- In symbolic logic, every statement can be denoted by a symbol.
- Some examples:
  - 2+2 = 5 is a statement because it is a false declaration.
  - Orange juice contains vitamin C is a statement that is true.
  - Open the door. This is not considered a statement since we cannot assign a true or false value to this sentence. It is a command, but not a statement.

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Statements in Logic

- A statement is called simple if it does not contain other statements.
- A statement is called composed if it uses logical connectives, such as “not”, “and”, “or”, etc.
- Some examples:
  - “It is raining today” is a simple statement.
  - “Today is either Monday or Tuesday” is a composed statement: “Today is Monday” or “Today is Tuesday”.
  - “Today is not Tuesday” is simple or composed?

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Exercise Questions

Decide if the following sentences are statements or not. If they are, use propositional symbols and logic connectives to express them.

- If I win the lottery, I'll be poor.
- The coach shouts, "run fast!"
- Today is the first day of the school.
- Today is either sunny or cloudy.

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Another Example

- "Either the housemaid or the butler killed Mr. X. However, if the housemaid would have done it, the alarm would have gone off, and the alarm did not go off. Therefore, the butler did it."

- The example has three statements (sentences).
- What are simple statements in them?
  - "the housemaid killed Mr. X. "
  - "the butler killed Mr. X. "
  - "the alarm went off."

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Using Symbols for Statements

- Use symbols, called propositional variables, to represent propositions:
  - H: The housemaid did it
  - B: The butler did it
  - A: The alarm went off
- Use logic connectives (or logic operators) to represent complex statements:
  - $H \vee B$: The housemaid or the butler did it
  - $H \rightarrow A$: If the housemaid did it, the alarm would go off
  - $\neg A$: The alarm did not go off

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Natural language is very ambiguous

- **The Sophist's Paradox**: A Sophist is sued for not paying his tuition by the school that educated him. He argued that he must win, since, if he loses, the school didn't educated him well enough, and doesn't deserve the money. The school argued that he must loss, since, if he win, he was educated well enough therefore should pay for it.
- What went wrong here?

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Symbols Make Things Clear

- **The Sophist's Paradox**: A Sophist is sued for not paying his tuition by the school that educated him. He argued that he must win, since, if he loses, the school didn't educated him well enough, and doesn't deserve the money. The school argued that he must loss, since, if he win, he was educated well enough therefore should pay for it.
- Propositions:
  - A: Sophist should pay the tuition
  - B: Sophist wins the law suit
  - C: Sophist was well educated in school
- Premises:
  - $C \rightarrow B$ If he was educated well, he will win
  - $B \rightarrow \neg A$ If he wins, he should not pay the tuition
  - $C \rightarrow A$ If he was educated well, he should pay
- The premises are inconsistent.

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Propositional and Boolean Logics

- Propositional logic is the study of propositional variables and logic operators.
- Propositional variables are called Boolean variables, in memory of logician George Boole.
- Boolean variables take two Boolean values: true and false (or 0 and 1, $\perp$ and $\top$, yes and no)
- Boolean logic includes any logic in which only Boolean values are considered.

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Proofs in Logic

- Transform symbolic representations using basic rules that reflect valid inferences:

1.	$H \vee B$	Assumption
2.	$H \rightarrow A$	Assumption
3.	$\neg A$	Assumption
4.	$\neg H$	2, 3 ***modus tollens***
5.	B	1, 4 ***disjunction law***

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Logics in Computer Science

- Propositional Logic
- First Order Logic
- Probability Logic
- Higher Order Logic
- Modal Logic
- Temporal Logic
- Hoare Logic

Applications:
- Constraint Satisfaction
- Database Design
- Programming Language
- Design Verification

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Uses of Formal Logic

- *Evaluation/Checking*:
  - Formal logic can be used to evaluate the validity of arguments.
- *Clarification/Specification*:
  - Formal logic can be used to express things in a precise and unambiguous way.
- *Demonstration/Proof*:
  - Formal logic can be used to figure out what follows from a set of assumptions.
- *Computation/Automated Reasoning*:
  - Formal logic can be used for machine reasoning.

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