Huffman coding

An exercise in the use of priority queue

Symbol	Frequency	Symbol	Frequency	Symbol	Frequency
space	186	b	47	g	15
e	103	d	32	p	15
t	80	l	32	b	13
a	64	u	23	v	8
o	63	c	22	k	5
i	57	f	21	j	1
n	57	m	20	q	1
s	51	w	18	x	1
r	48	y	16	z	1

Source: Donald Knuth, *The Art of Computer Programming* (Volume 3) p 441

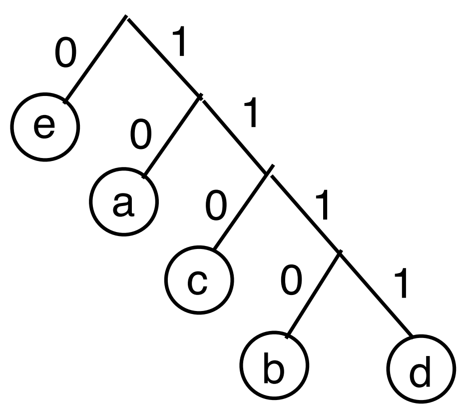
This table shows the average occurrence of individual letters in every 1000 letters. Each letter can be encoded by a custom binary code.

The problem. How will you encode these symbols so that the binary file has the smallest size?

Straight ASCII (that will use n bytes for coding n characters is not the optimal solution, when you know the frequencies of these characters. An *efficient solution* will use only a few bits for the frequently used characters, but may use more bits to encode less frequently used characters. Of course this will be a custom encoding scheme.

[Application: saving the transmission bandwidth]

A naïve solution is as follows. Suppose the frequencies of the following five letters satisfy the order e > a> c > b > d



Here are the codes:

e = 0, a = 10, c = 110, b= 1110, d = 1111

This is ok, but may not be optimal when the frequencies are known.

A smaller scale example

e	r	s	t	n	l	z	x
34	22	24	28	15	10	9	8

Frequency in an average sample of size 150 letters

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Enqueue these in a priority queue


Dequeue (the letter/subtree with smallest count)
Dequeue (the letter/subtree with smallest count)
Form a subtree by adding a common parent to the
above two and enqueue into the priority queue again


Repeat these steps till a binary tree is formed.
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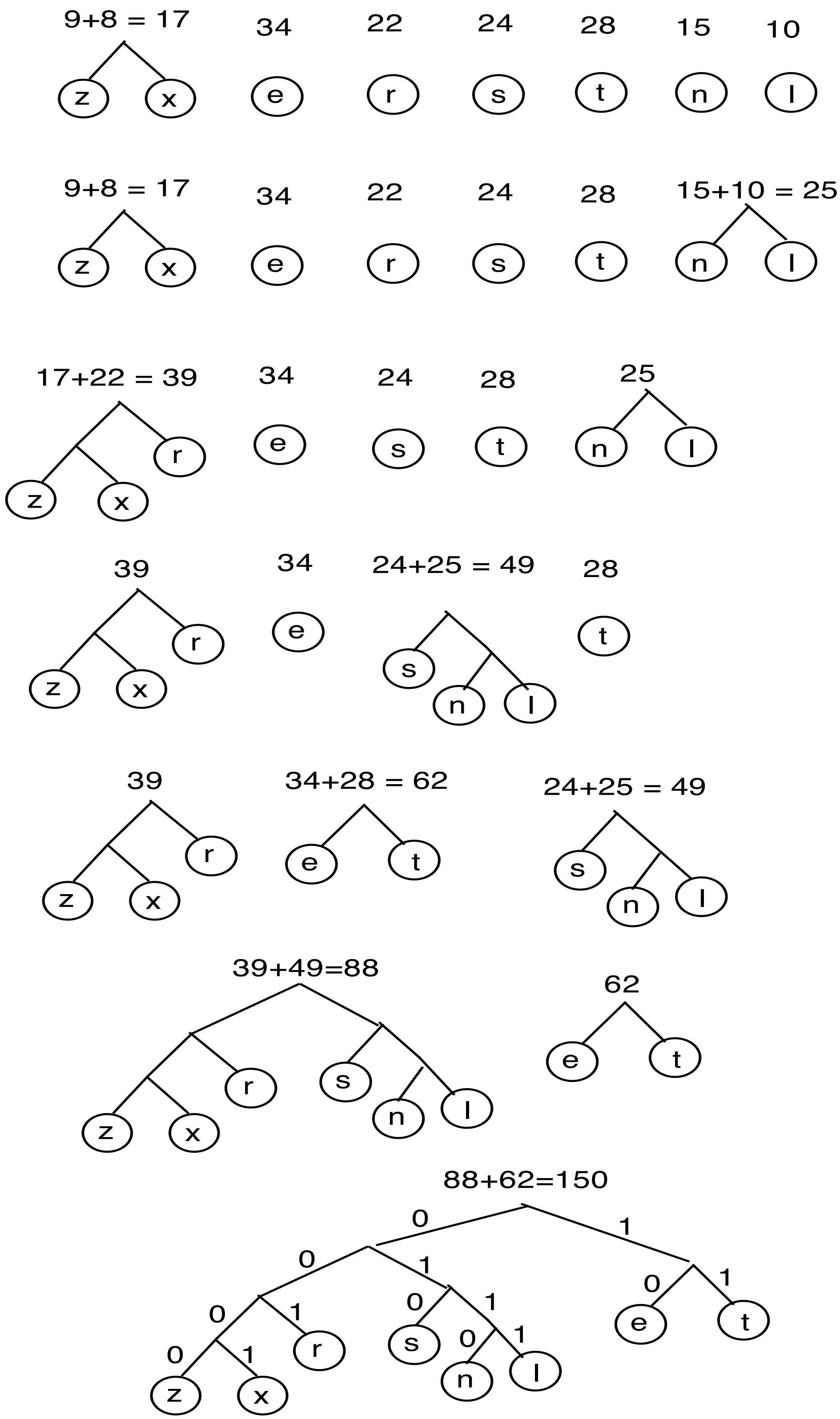
The tree is shown in the next page. This leads to the following codes.

z = 0000	n = 0110
x = 0001	l = 0111
r = 001	e = 10
s = 010	t = 11


9+8 = 17
34
22
24
28
15
10
z x e r s t n l
9+8 = 17
34
22
24
28
15+10 = 25
17+22 = 39
34
24
28
25
39
34
24+25 = 49
28
39
34+28 = 62
24+25 = 49
39+49=88
62
88+62=150
0 1 0 1 0 1 0 1 0 1 0 1 0 1