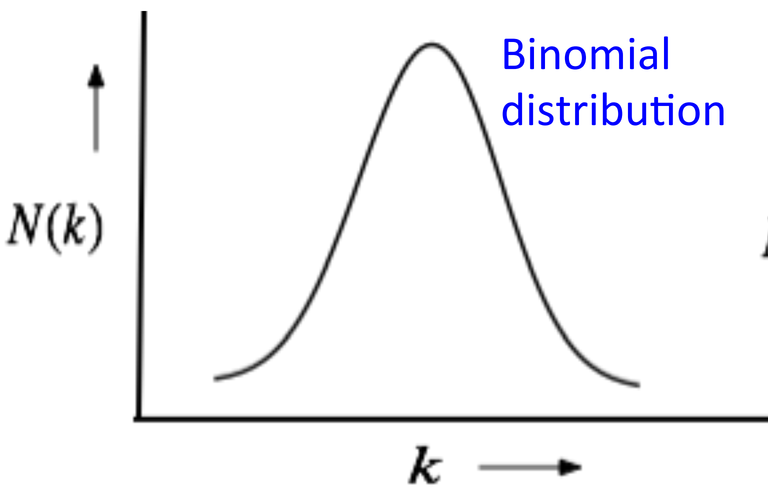
Peer-to-Peer and Social Networks

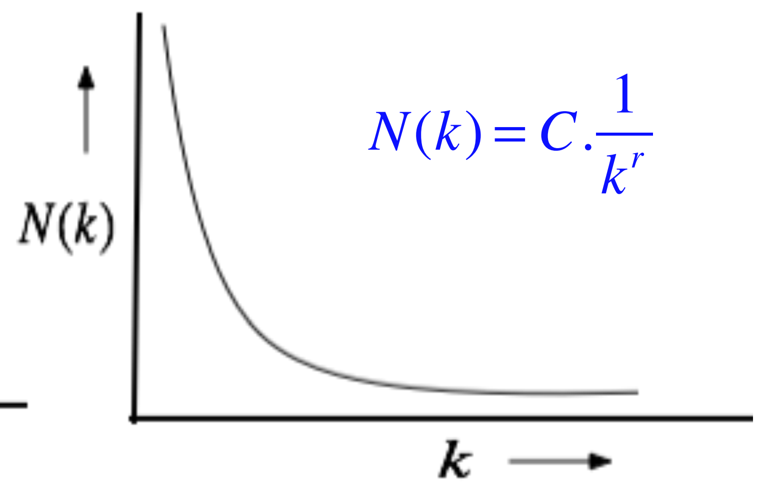
Power law graphs

Random vs. Power-law Graphs

The degree distribution in of the webpages in the World Wide Web follows a power-law

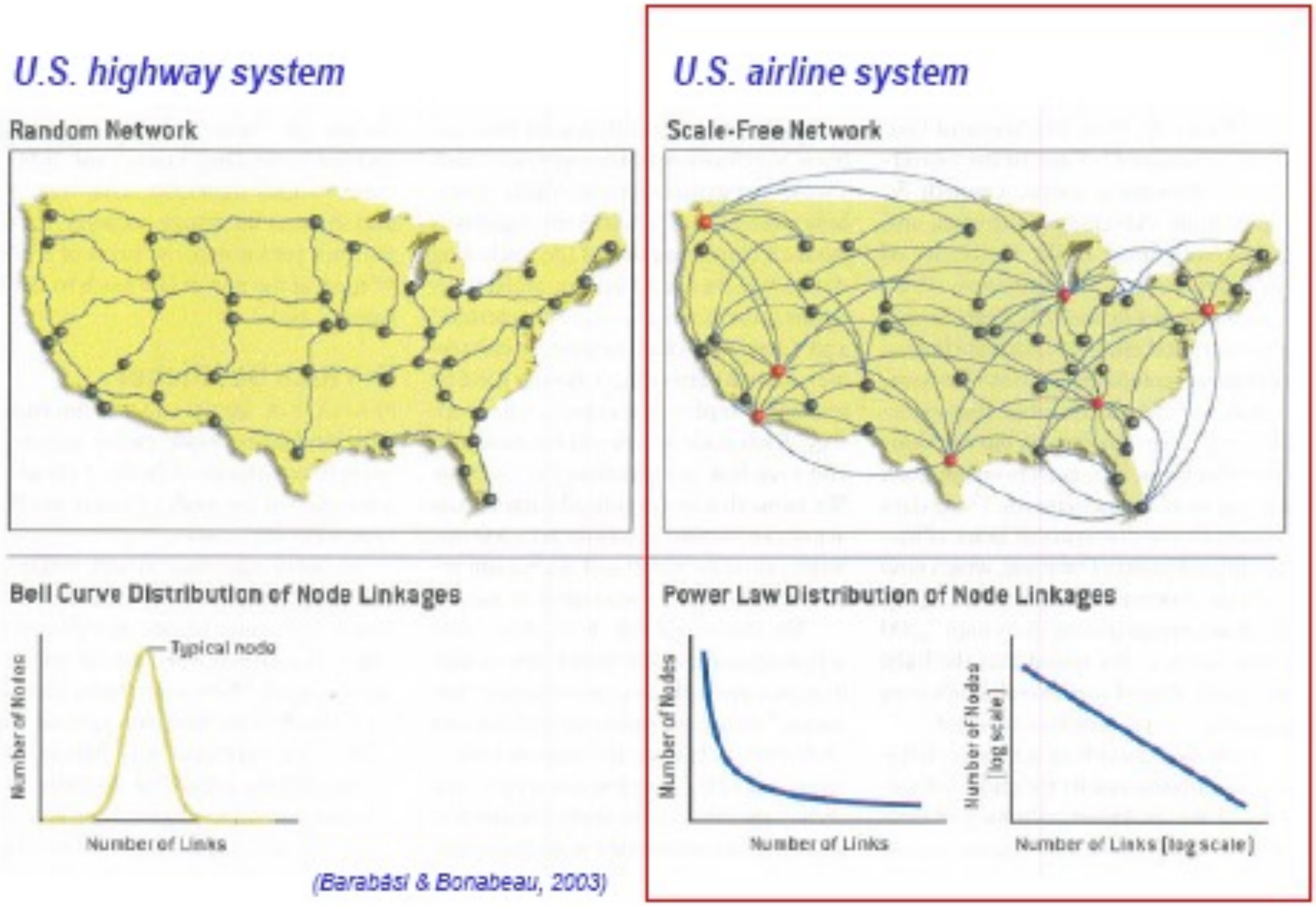


Random graph



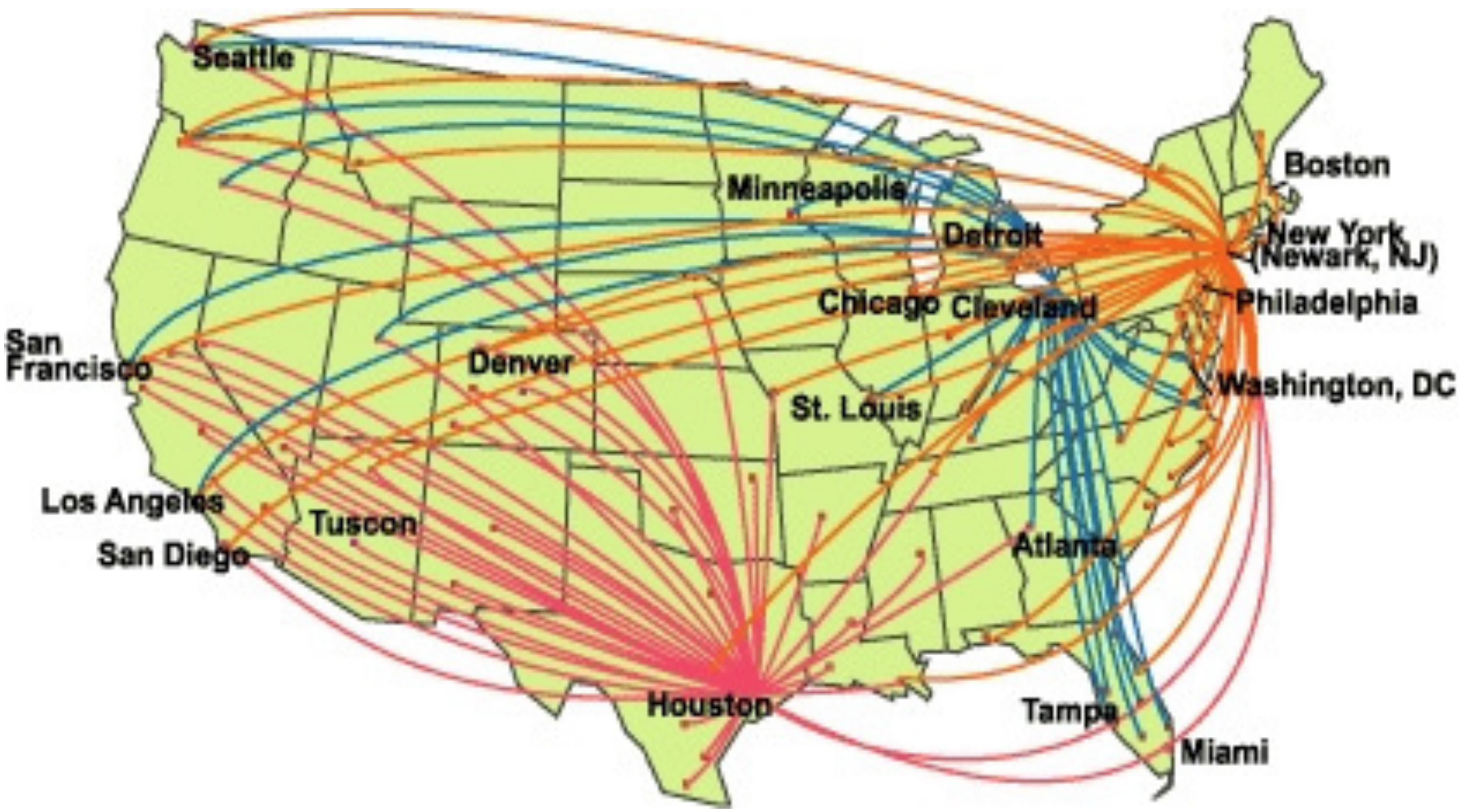
PowerLaw distribution

Random vs. Power-Law networks



(Barabási & Bonabeau, 2003)

Example: Airline Routes



Think of how new routes are added to an existing network

Examples of Power law distribution

Also known as **scale-free graph**. Other examples are

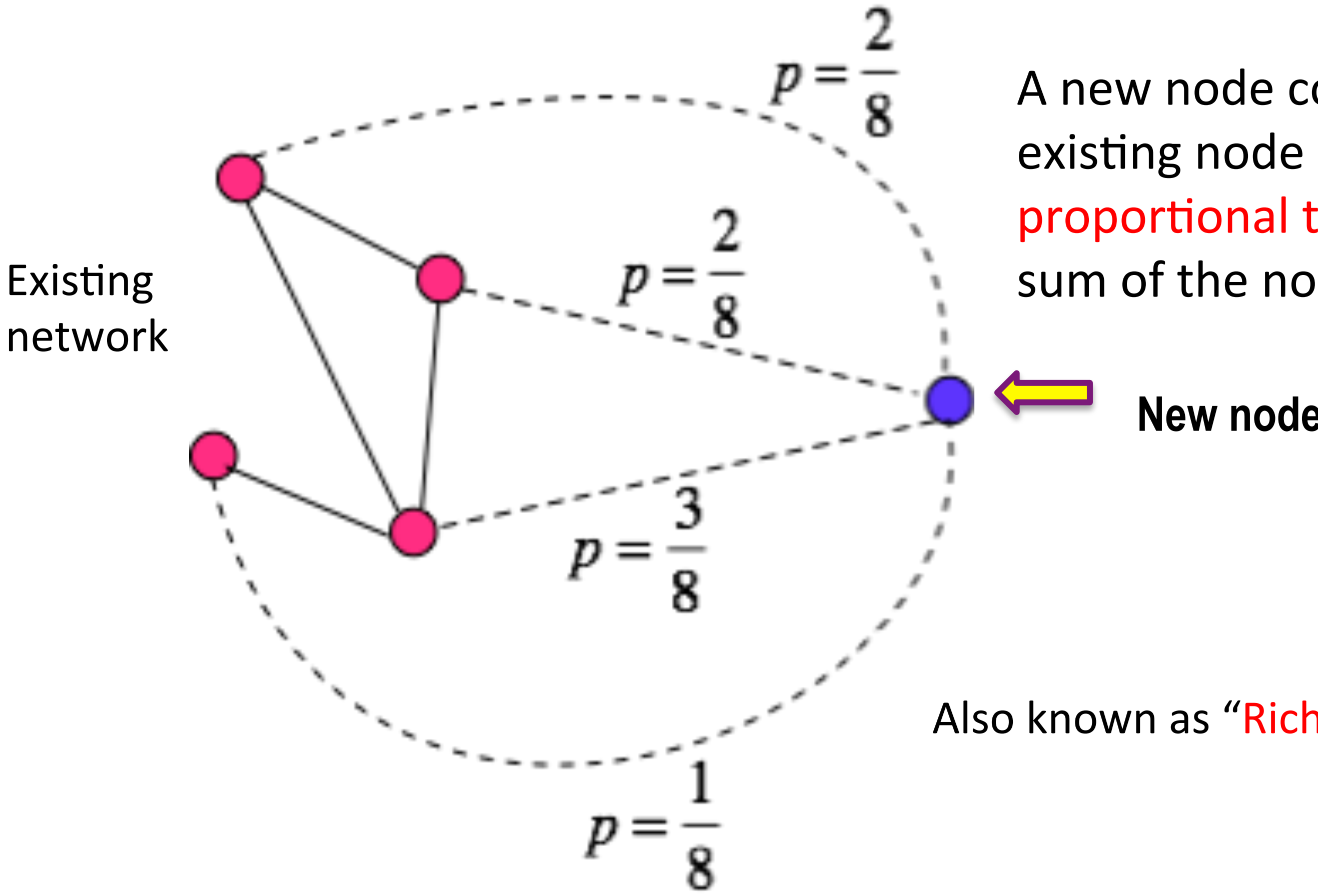
-- Air route graph

-- Income and number of people with that income

-- Magnitude and number of earthquakes of that magnitude

-- Population and number of cities with that population

Preferential attachment



A new node connects with an existing node with a probability proportional to its ***degree***. The sum of the node degrees = 8

Also known as "Rich gets richer" policy

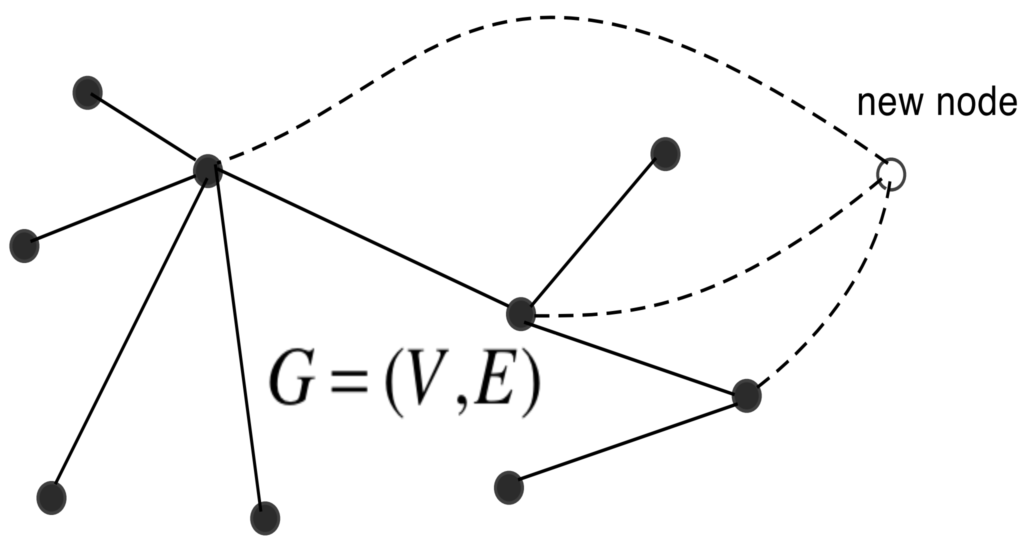
This leads to a power-law distribution (Barabási & Albert)

Preferential attachment

Barabási and Albert showed that when large networks are formed by the rules of preferential attachment , the resulting graph shows a power-law distribution of the node degrees.

We will derive it in the class, so follow the lecture.

Preferential attachment



At t = 0, there are no nodes.

At t = 1, one node appears.

Thereafter, each time unit,

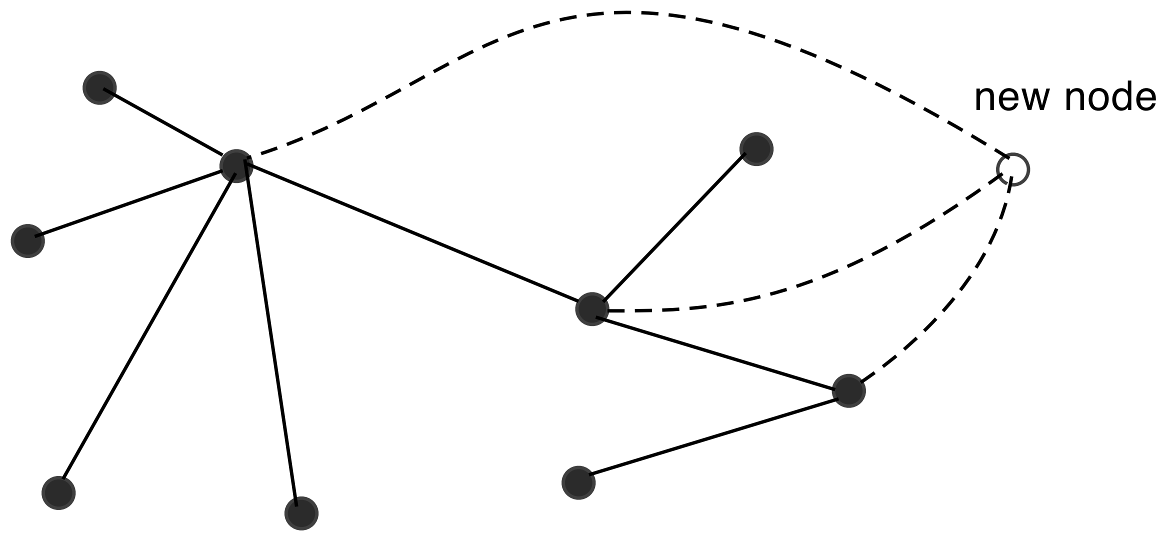
a new node is added

Degree of node i = $\delta(i)$

The probability that the new node connects with an existing node i = $C.\ \delta(i)$

Since $\sum_{i \in V} C.\ \delta(i) = 1$ and $\sum_{i \in V} \delta(i) = 2\,|V|$ so $C = \frac{1}{2t}$

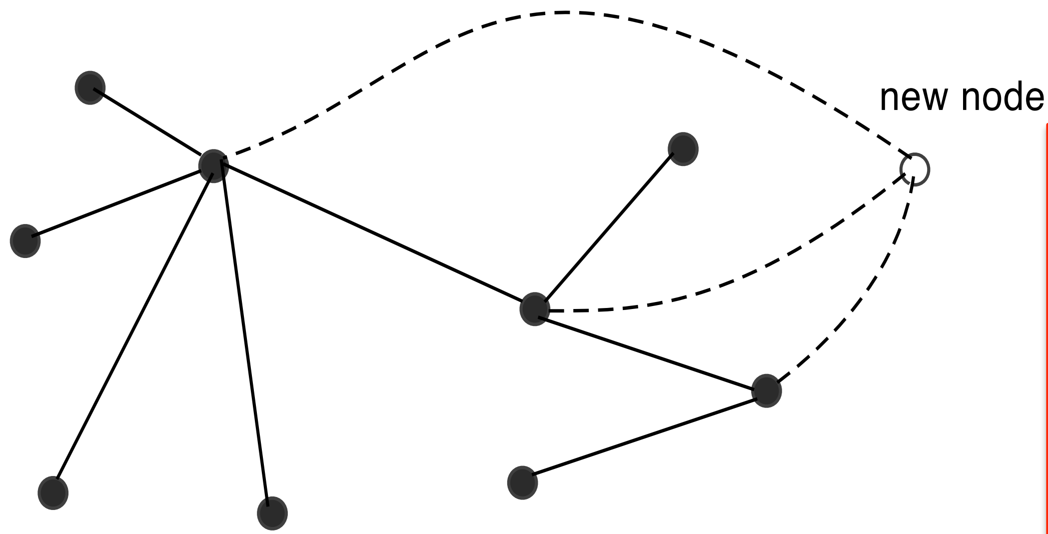
Preferential attachment



$n(k,t)$ = number of nodes with degree k after time step t

$$n(k,t+1) = n(k,t) + n(k-1,t).\frac{k-1}{2t} - n(k,t).\frac{k}{2t}$$

Preferential attachment



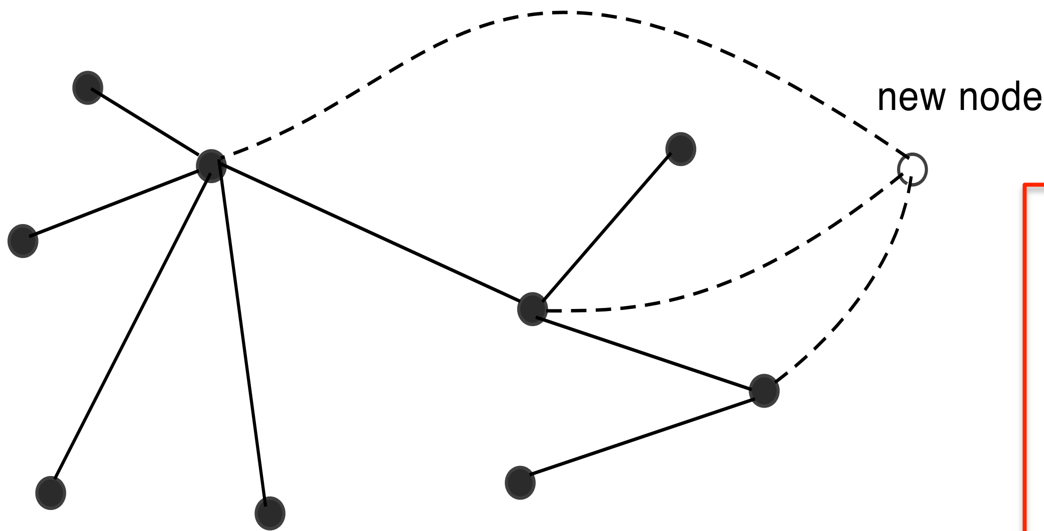
$$f(k,t) = \frac{n(k,t)}{|V|}$$

is then fraction of nodes with degree k at time t

$$n(k,t+1) = n(k,t) + n(k-1,t).\frac{k-1}{2t} - n(k,t).\frac{k}{2t}$$

$$(t+1).\ f(k,t+1) = t.\ f(k,t) + \frac{1}{2}\left[(k-1).\ f(k-1,t) - k.\ f(k,t)\right]$$

Preferential attachment



As $t \longrightarrow \infty$,

$$f(k,t+1) \rightarrow f(k,t)$$

Call it $f(k)$

$$(t+1).\ f(k,t+1) = t.\ f(k,t) + \frac{1}{2}\left[(k-1).\ f(k-1,t) - k.\ f(k,t)\right]$$

$$f(k) = \frac{1}{2}\left[(k-1).\ f(k-1) - k.\ f(k)\right]$$

$$f(k) = \frac{k-1}{k+2}.\ f(k-1)$$

Preferential attachment

$$f(k)=\frac{k-1}{k+2}.\ f(k-1)$$

$$\frac{k-1}{k+2}.\frac{k-2}{k+1}.\frac{k-3}{k}, \frac{k-4}{k-1}.....\frac{4}{7}.\frac{3}{6}.\frac{2}{5}.\frac{1}{4}\ f(1)$$

$$\frac{3.2.1}{(k+2).(k+1).\ k}.\ f(1)$$

$$f(k)=\frac{4}{k(k+1)(k+2)}$$

$f(k)$ is of the order of $\frac{1}{k^3}$

$$n(1,t+1)=n(1,t)+1\overset{*}{-}n(1,t).\frac{1}{2t}$$

$$(t+1).\ f(1,t+1)=t.\ f(1,t)+1-\frac{f(1,t)}{2}$$

$$t\rightarrow\infty,\ f(1,t+1)=f(1,t)=f(1)$$

$$f(1)=\frac{2}{3}$$

* Before time step (t+1), the new node is the only node with degree 0, and its degree will change to 1

Other properties of power law graphs

- Graphs following a power-law distribution $N(k) \sim k^{-r} (2 < r < 3)$ have a small diameter $d \sim \ln \ln n$ (n = number of nodes).
- The clustering coefficient decreases as the node degree increases (power law again)
- Graphs following a power-law distribution tend to be highly resilient to random edge removal, but quite vulnerable to targeted attacks on the hubs.