Notes on Predicate Logic
Basic Definitions & Examples

Here is a brief synopsis of the predicate logic definitions together with a few illustrative examples.

Terms
Atomic terms are either variables or constant names. **Compound terms** are function names applied to other (possibly compound) terms.

Formulas
Atomic formulas are predicate names applied to argument *terms*. **Compound formulas** are atomic formulas joined together with the usual propositional logical connectives, plus *universal quantification* (∀) and *existential quantification* (∃).

Interpretations
An *interpretation* identifies:
- a universe to be used in universal and existential quantification,
- an assignment of each function name to some function acting on the universe,
- an assignment of each predicate name to some relation over the universe,
- an assignment of each free variable and each constant name to some value in the universe .

Formula evaluation
Once an interpretation makes assignments to all the names appearing in a formula, evaluation can proceed in the expected manner and results in a true or false value.

Valid formulas and Models
An interpretation is said to **satisfy** a formula if the formula evaluates to true in that interpretation, and we say the interpretation is a **model** of the formula. A formula F that is satisfied by *every* interpretation is called **valid**, written $\models F$.

Examples
At the outset we need to identify each of the names we will use in a logical system, and for functions and predicates, the number of arguments (f/3 means f takes 3 arguments).

Function names: a/1, b/2.
Constant name: c.
Variable names: x, y.
Predicate names: p/1, q/2.
Atomic terms: c, x, y.
Compound terms: a(c), a(x), b(x,y), b(a(c),y), b(b(x,y), b(a(c),y)).
Atomic formulas: p(a(x)), p(b(b(x,y),b(a(c),y))), q(b(a(c),y), b(x,y)).
Compound formulas: b(x,y) □ b(a(c),y), a(x) □ b(x,y), □x p(a(x)), □x(□y b(x,y)).
Interpretation $\mathcal{I}$: universe = Natural numbers; $\mathcal{I}$(c) = 2; $\mathcal{I}$(x) = 5; $\mathcal{I}$(y) = 21; $\mathcal{I}$(a(n)) = 2*n; $\mathcal{I}$(b(m,n) = m+n; $\mathcal{I}$(p(n)) = n>7; $\mathcal{I}$(q(m,n)) = m–1>n. So $\mathcal{I}$ is a model of p(a(x)), but does not satisfy q(a(c), b(c,c)).