Example program proof — conditional rule

In this example, we prove a program assertion establishing partial correctness of a program fragment computing the absolute value of a number.

$\{$ **true** $\}$

if $X < 0$ **then** $A := -X$ **else** $A := X$

$\{ (X < 0 \wedge A = -X) \vee (X \geq 0 \wedge A = X) \}$

1. $\vdash \{ (X < 0 \wedge -X = -X) \vee (X \geq 0 \wedge -X = X) \}$

    $A := -X$

    $\{ (X < 0 \wedge A = -X) \vee (X \geq 0 \wedge A = X) \}$

    by the assignment axiom.

2. The pre-condition in step 1 is logically equivalent to $X \leq 0$, and

    $\vdash X < 0 \Rightarrow X \leq 0$ so by strengthening the pre-condition of step 1

    $\vdash \{ X < 0 \} A := -X \{ (X < 0 \wedge A = -X) \vee (X \geq 0 \wedge A = X) \}$

3. $\vdash \{ (X < 0 \wedge X = -X) \vee (X \geq 0 \wedge X = X) \}$

    $A := X$

    $\{ (X < 0 \wedge A = -X) \vee (X \geq 0 \wedge A = X) \}$

    by the assignment axiom, and this pre-condition is logically equivalent to $X \geq 0$.

4. Using steps 2 and 3 with the conditional rule proves the program.