22C:34 Exam #3

Spring 2003

Good luck!

1. [20 pts] Expand $(3x - 2y)^3$.

2. [20 pts] Use the binomial theorem to show that: $\sum_{k=0}^{n} \binom{n}{k} 2^k = 3^n$.

3. [20 pts] Find the coefficient of $x^2 y^3 z^4$ in the expansion of $(x+y+z)^9$.

4. [20 pts] Determine whether each relation defined on the set of positive integers is reflexive, symmetric, antisymmetric, transitive, and/or a partial order:

   a) $(x,y) \in R$ if $x = y^2$

   b) $(x,y) \in R$ if $x > y$

   c) $(x,y) \in R$ if 3 divides $x - y$

   d) $(x,y) \in R$ if $x = y$

5. [10 pts] Let X = { 1, 2, 3, 4, 5 }, Y = { 3, 4 }, and C = { 1, 3 }. Define R on $\rho(X)$, the power set of X, as A R B if and only if A U Y = B U Y. Show that R is an equivalence relation.

6. [10 pts] By drawing a digraph, give an example of an equivalence relation on { 1, 2, 3, 4, 5, 6 } having exactly 4 equivalence classes.

7. [20 pts] Draw the Hasse diagram for the partial ordering x divides y on the set { 2, 3, 6, 9, 12, 18, 27 }.

8. [15 pts] Give an example of a function that

   a) is 1-1 but not onto
   b) is onto but not 1-1
   c) is neither 1-1 nor onto

9. [20 pts] Let f: $S \to T$ and $g : T \to U$ be functions. Find an example where g o f is 1-1 but g is not 1-1.

10. [20 pts] Find the composition of the following cycle representing a permutation on A = { 1, 2, 3, 4, 5, 6, 7, 8 }. Write your result as the composition of disjoint cycles.