22C:34 Exam #2

Spring 2003

Good luck!

1. [10 pts] Write the first five values in the sequence:

$$\begin{aligned} D(1) &= 3 \\ D(2) &= 5 \\ D(n) &= (n-1)D(n-1) + (n-3)D(n-2),\ \ n \geq 3. \end{aligned}$$

2. [10 pts] A collection W of strings of symbols is defined recursively by:

   1. $a, b,$ and c belong to W
   2. If X belongs to W, so does $a(bXc)a$

   List 2 strings of length 7 and 2 strings of length 13; string length includes the parens.

3. [20 pts] A sequence is recursively defined by

$$\begin{aligned} S(0) &= 1 \\ S(1) &= 1 \\ S(n) &= 2S(n-1) + S(n-2), for\ n \geq 2 \end{aligned}$$

   Prove that $S(n)$ is an odd number for $n \geq 0$.

4. [20 pts] Solve the recurrence relation

$$P(n) = 2P(n-1) + n2^n \text{ for n} \geq 2$$

   subject to the basis step

$$P(1) = 2$$

   Recall the solution formula

$$S(n) = c^{n-1}S(1) + \sum_{i=2}^{n} c^{n-i}g(i)$$

   for a first-order recurrence relation $S(n) = cS(n-1) + g(n)$ with constant coefficients.

5. [30 pts] A recurrence relation of the form

$$S(n) = cS(\frac{n}{2}) + g(n) \text{ for n} \geq 2, n = 2^m$$

has a solution of

$$S(n) = c^{\log n} S(1) + \sum_{i=1}^{\log\ n} c^{(\log\ \mathrm{n})-i} g(2^i)$$

Solve

$$\begin{aligned} P(1) &= 1 \\ P(n) &= 2P(\frac{n}{2}) + n^2 \end{aligned}$$

6. [20 pts] Find $P(S)$, the power set of S, for $S = \{\varphi, \{\varphi\}, \{\{\varphi\}\}, \{\{\varphi, \{\varphi\}\}\}\}$.

7. [15 pts] A palindrome is a string of characters that reads the same forward and backward. How many five-letter English language palindromes are possible?

8. [20 pts] A group of students plan to order pizza. If 13 will eat mushroom topping, 10 will eat green pepper, 12 will eat extra cheese, 4 will eat both mushroom and green pepper, 5 will eat both green pepper and extra cheese, 7 will eat both mushroom and extra cheese, and 3 will eat all three toppings, how many students are in the group?

9. [15 pts] A soccer team carries 18 players on the roster; 11 players make a team. In how many ways can a team be chosen?

10. [15 pts] Explain why $\binom{n}{n-1} = \binom{n}{1}$.