22C:34 Exam #1

Spring 2003

Good luck!

1. [10 pts] Construct a truth table for (A $\rightarrow$ B) $\leftrightarrow$ A$'\vee$ B
2. [10 pts] Use propositional logic to prove: (P $\vee$ Q) $\wedge$ P$'$ $\rightarrow$ Q.
3. [10 pts] Identify the scope of each of the quantifiers and free variables in ($\exists x$) ($\exists y$) [A(x,y) $\wedge$ B(y,z) $\rightarrow$ A(a,z)].
4. [10 pts] Give an interpretation to prove that ($\exists x$) A(x) $\wedge$ ($\exists x$) B(x) $\rightarrow$ ($\exists x$) [A(x) $\wedge$ B(x)] is not valid.
5. [10 pts] Prove that ($\exists x$) [A(x) $\wedge$ B(x)] $\rightarrow$ ($\exists x$) A(x) $\wedge$ ($\exists x$) B(x) is a valid argument.
6. [10 pts] Prove: the square of an odd integer equals 8 k + 1 for some integer k.
7. [20 pts] Prove that for any positive integer n, $2^{2n}-$ 1 is divisible by 3.
8. [20 pts] Prove: $a + ar + ar^2 + ... + ar^{n-1} = \frac{a-ar^n}{1-r}$, $r \neq 1$, $n \geq 1$.